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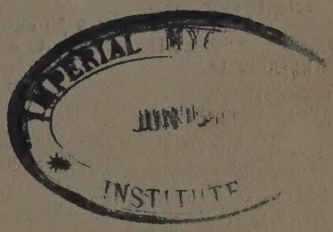

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TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

J. Dubnov et **M. Sabyrov.** Les tenseurs fondamentaux dans la théorie métrique des congruences de sphères 615

Dsh. H. Karimov. On the Periodical Solutions of Non-linear Equations of the Fourth Order 618

L. Keldych. Sur les transformations ouvertes des ensembles A 622

A. Norden. On Pairs of Conjugate Parallel Translations in n-dimensional Spaces 625

MÉCANIQUE

A. Sternfeld. On Fuel Consumption by a Rocket Crossing the Atmosphere with Constant Acceleration 629

THÉORIE DE L'ÉLASTICITÉ

S. Michlin. On the Fundamental Problem in the Theory of Mining Pressure 633

PHYSIQUE MATHÉMATIQUE

D. J. Scherman. On Poincaré's Problem in the Theory of Potential 636

PHYSIQUE

A. Andronov and **G. Gorelik.** On Resonance Phenomena Associated with the Movement of a Relativistic Particle in the Cyclotron 640

PHYSIQUE DU GLOBE

N. N. Kalitin. Theoretical and Actual Total Heat of Solar Radiation for Cloudless Sky 643

PHYSIQUE APPLIQUÉE

F. A. Korolev. A Spark Generator with Forced Ignition for Spectroscopic Analysis 646

CHIMIE

L. G. Berg. On Area Measurements in Thermograms for Quantitative Estimations and the Determination of Heats of Reaction 648
S. L. Lelchuk. The Effect of Promoters in the Catalytic Dehydrogenation of Alcohols to Esters as Related to Their Place in the Periodical Table . 652

CHIMIE PHYSIQUE

A. A. Balandin, Corresponding Member of the Academy, and **J. T. Eidus.** On the Rôle of the Principle of Conservation of the Valence Angle in the Multiplet Theory of Catalysis 655
M. A. Sadovsky. On the Inner Structure of a Cumulative Jet 659

CHIMIE DU GLOBE

A. B. Ronov, M. E. Vladimirova, N. I. Polevaya and **F. E. Starik.** On the Geochemistry of the Carboniferous and Permian Deposits of the Chistopol Region of Tataria 662

CHIMIE BIOLOGIQUE

B. A. Rubin, O. I. Pushkinskaya and **V. E. Sokolova.** On the Biochemical Characteristics of the Resistance of Plants to Microorganisms 665

PÉTROGRAPHIE

V. N. Kotliar. On Orbicular Gabbros met near the Voskresenka Village, Armenian SSR 669

PALÉONTOLOGIE

A. G. Vologdin, Corresponding Member of the Academy, and **C. B. Cordet.** On a Poorly Studied but Important Group of Fossil Organisms 671

MICROBIOLOGIE

M. V. Fedorov. Fixation of Atmosphere Nitrogen by *Azotobacter* as Influenced by the Substances Reacting with the Amino-group of the Catalyst 675

PHYSIOLOGIE VÉGÉTALE

M. Moissejewa. A Contribution to the Study of the Function of the Chlorophyll-bearing Tissue in the Conductive Bundles of the Stem in *Cucurbita Pepo* 679

PHYSIOLOGIE

E. F. Kisseleva. A Contribution to the Knowledge of Chemical Relations between Plants and Animals 682

MORPHOLOGIE EXPÉRIMENTALE

N. V. Belsky. Correlation of Growth and Differentiation During the post-Embryonic Development of the Pigeon 684
V. V. Popov. Interrelations between the Inductions of Embryonic Type and Endocrine Developmental Factors 687

HELMINTHOLOGIE

K. I. Skrjabin, Member of the Academy, and **N. P. Shikhobalova.** A New Rearrangement of the Taxonomy of the Nematodes Belonging to the Family *Filariidae* Cobbold, 1864 690

Le présent numéro est suivi de l'indice au volume XLIX

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MATHÉMATIQUES

LES TENSEURS FONDAMENTAUX DANS LA THÉORIE MÉTRIQUE DES CONGRUENCES DE SPHÈRES

Par J. DUBNOV et M. SABYROV

(*Présenté par A. N. Kolmogoroff, de l'Académie, le 1. IX. 1945*)

En résumant les recherches de Ribaucour, Darboux et ses propres études, Bianchi (1) a construit une théorie métrique des congruences de sphères en partant des deux formes quadratiques binaires qui sont définies (d'une manière ambiguë) par la congruence. Cependant cette construction ne peut être considérée comme complète: on ne sait pas 1) en quelle mesure reste arbitraire le système des formes mentionnées, 2) en quelle mesure elles définissent la congruence (si l'on compare avec la théorie des surfaces ou bien des congruences rectilignes, on peut constater l'absence des conditions du type Gauss-Codazzi et du théorème du type de Bonnet).

Le but de la présente Note est de combler cette lacune.

1. Théorème fondamental. Supposons que pour la sphère $\sum(u^1, u^2)$ génératrice de la congruence

$$\mathbf{r}(u^1, u^2),\ \mathbf{R}(u^1, u^2),\ \mathbf{N}(u^1, u^2),\ \rho(u^1, u^2) \tag{1}$$

désignent respectivement: 1) le rayon vecteur du centre*; 2) le rayon vecteur du point où la sphère touche une des deux nappes (choisie d'une manière déterminée) de son enveloppe; 3) le vecteur unitaire de la normale à cette nappe au point considéré; 4) le rayon de la sphère pris avec son signe $\rho = (\mathbf{r} - \mathbf{R})\,\mathbf{N}$.

Appelons tenseurs de Bianchi de la congruence de sphères les tenseurs symétriques

$$G_{ij} = \mathbf{r}_i \mathbf{r}_j - \rho_i \rho_j, \quad B_{ij} = \mathbf{r}_i \mathbf{N}_j = \mathbf{r}_j \mathbf{N}_i \quad (i, j = 1, 2)$$

où $\mathbf{r}_i = \partial \mathbf{r}/\partial u^i$, et ρ_i, $\mathbf{N}_i$ ont des valeurs analogues; les deux formes quadratiques de Bianchi mentionnées ne diffèrent que par les notations $G_{\alpha\beta}du^\alpha du^\beta$ (carré de la «distance tangentielle» entre deux sphères infiniment voisines) et de $B_{\alpha\beta}du^\alpha du^\beta$ (seconde forme de Kummer pour le système formé de la surface des centres et de la congruence rectiligne des normales à la nappe choisie de l'enveloppe). Comme toujours, nous allons supposer que le tenseur G_{ij} ne dégénère pas; on exclut de cette manière le cas et ce cas seulement où les deux nappes de l'enveloppe coïncident entre elles ayant la surface des centres pour sa développée. Dans la suite la différentiation covariante (symbole ∇_i) est toujours effectuée par rapport au tenseur G_{ij}, et l'élévation et l'abaissement des indices s'effectue au moyen des tenseurs discriminants ε_{ij}, ε^{ij}, formés à

* On ne considère que les congruences de sphères pour lesquelles la «surface des centres» $\mathbf{r} = \mathbf{r}(u^1, u^2)$ ne dégénère pas.

partir de G_{ij}

$$\varepsilon_{11}=\varepsilon_{22}=0,\quad \varepsilon_{12}=-\varepsilon_{21}=\sqrt{\operatorname{Det} G_{ij}},\quad \varepsilon^{11}=\varepsilon^{22}=0,\quad \varepsilon^{12}=-\varepsilon^{21}=1:\sqrt{\operatorname{Det} G_{ij}}$$

suivant les règles

$$\varepsilon^{i\alpha}t_\alpha=t^i,\quad \varepsilon_{\alpha i}t^\alpha=t_i$$

On peut maintenant énoncer le

Théorème A. *Pour qu'il existe une congruence de sphères dont le rayon* ρ *et les tenseurs de Bianchi* G_{ij}, B_{ij} *sont donnés, il est nécessaire et suffisant que les trois conditions suivantes soient vérifiées*

$$\left.\begin{aligned}&(P,B)\,\rho_i+G_{i\alpha}\nabla_\beta B^{\alpha\beta}=0\quad (i=1,\ 2)\\&(1+\Delta\rho)\,(B,B)-E^{\alpha\beta}\nabla_\alpha\rho_\beta=\overset{G}{K}\end{aligned}\right\}\tag{2}$$

où $(BB)=\frac{1}{2}B^{\alpha\beta}B_{\alpha\beta}$, $\Delta\rho=G^{\alpha\beta}\rho_\alpha\rho_\beta$, $\overset{G}{K}$ *est la courbure de la forme* $G_{\alpha\beta}du^\alpha du^\beta$. *Si ces conditions sont vérifiées, la congruence est déterminée à un mouvement rigide près.*

Nous supposons maintenant que le tenseur P_{ij} ne dégénère pas non plus [$(B,B)\neq 0$; on exclut ainsi les congruences et ces congruences seulement, pour lesquelles la nappe de l'enveloppe, qui figure dans la construction, est une surface développable]; nous pouvons éliminer ρ de (2). En remarquant que la première des conditions (2) détermine ρ à une constante additive près, on voit que les sphères de la congruence sont déterminées à une «dilatation» (au sens de Lie) près. Cela étant, on peut énoncer le théorème suivant qui concerne les tenseurs de Bianchi seulement.

Théorème A'. *Pour qu'il existe une congruence à tenseurs de Bianchi* G_{ij}, B_{ij} *qui ne dégénèrent pas, il est nécessaire et suffisant que les deux conditions suivantes soient vérifiées*

$$\left.\begin{aligned}&G^\mu_\beta\nabla_\mu\frac{\nabla_\alpha B^{\alpha\beta}}{(B,B)}=0,\quad \left(\frac{G_{i\beta}\nabla_\alpha B^{\alpha\beta}}{(B,B)}=\operatorname{grad}\right)\\&(P,B)+\frac{1}{(B,B)}G_{\lambda\beta}\nabla_\alpha B^{\alpha\beta}\nabla_\gamma B^{\gamma\lambda}+B^{\mu\gamma}G_{\beta\gamma}\nabla_\mu\frac{\nabla_\alpha B^{\alpha\beta}}{(B,B)}=\overset{G}{K}\end{aligned}\right\}\tag{3}$$

Si ces conditions sont vérifiées, la congruence est déterminée à un mouvement rigide suivi d'une dilatation arbitraire près.

2. Les tenseurs de Gauss. Congruence de Ribaucour. Une autre manière de définir une congruence de sphères consiste à considérer comme connus outre le ρ les tenseurs de Gauss de l'une des deux nappes de l'enveloppe [2]; on emploie ici et dans la suite les notations de la Note [2]:

$$G_{ij}=\mathbf{R}_i\mathbf{R}_j,\quad B_{ij}=-\mathbf{R}_i\mathbf{N}_j{}^*$$

Il existe entre les tenseurs G_{ij}, B_{ij} et G_{ij}, B_{ij} une réciprocité remarquable exprimée par les formules (on suppose que le tenseur G_{ij} ne dégénère pas)

$$\left.\begin{aligned}&G_{ij}=\tilde{G}^{\alpha\beta}(\rho B_{i\alpha}-G_{i\alpha})(\rho B_{j\beta}-G_{j\beta})\\&B_{ij}=\tilde{G}^{\alpha\beta}B_{i\alpha}(\rho B_{j\beta}-G_{j\beta})\end{aligned}\right\}\tag{4}$$

$$\left.\begin{aligned}&G_{ij}=\tilde{G}^{\alpha\beta}B_{i\alpha}(\rho B_{i\alpha}-G_{i\alpha})(\rho B_{j\beta}-G_{j\beta})\\&B_{ij}=\tilde{G}^{\alpha\beta}B_{i\alpha}(\rho B_{j\beta}-G_{j\beta})\end{aligned}\right\}\tag{5}$$

où $\tilde{G}^{ij}$ et $\tilde{G}^{ij}(=G^{ij})$ sont les tenseurs réciproques à G_{ij} et G_{ij} respectivement (par exemple, $\tilde{G}^{i\alpha}G_{k\alpha}=\delta^i_k$).

* Comme les vecteurs N et—N peuvent être échangés, les tenseurs B_{ij}, B_{ij} et ρ doivent être considérés comme déterminés à un facteur commun près égal à ± 1.

Cette réciprocité algébrique entre les couples G_{ij}, B_{ij} et G_{ij}, P_{ij} ne permet cependant pas d'espérer que chaque congruence à tenseurs de Bianchi G_{ij}, B_{ij} peut être transformée en une congruence à tenseurs de Bianchi G_{ij}, B_{ij}, car les relations différentielles (3) entre les tenseurs G_{ij}, B_{ij} ne sont pas les mêmes que pour les tenseurs G_{ij}, B_{ij} (les équations de Gauss-Codazzi). Les relations (2) permettent facilement de voir que G_{ij} et B_{ij} ne vérifient les conditions de Gauss-Codazzi que dans les deux cas suivants: 1) pour $\rho = \text{const}$, c'est-à-dire dans le cas géométriquement évident, où la surface des centres est parallèle aux deux nappes de l'enveloppe; 2) si G_{ij} et B_{ij} sont les tenseurs de Gauss d'une certaine surface développable, et ρ vérifie l'équation $B^{\alpha\beta}\nabla_\alpha\rho_\beta = 0$.

Signalons encore une relation algébrique entre les tenseurs de Bianchi et ceux de Gauss

$$\tilde{G}^{i\alpha}B_{k\alpha} = \frac{\rho K\delta^i_k - \tilde{G}^{i\alpha}B_{k\alpha}}{1 - 2\rho H + \rho^2 K} \tag{6}$$

où K et H sont respectivement la courbure de Gauss et la courbure moyenne de la nappe de l'enveloppe.

Comme application, considérons la congruence de sphères de Ribaucour; on sait qu'elle est caractérisée par le fait que les deux nappes de l'enveloppe se correspondent par les lignes de courbure (si l'on considère comme correspondants sur deux nappes les points qui appartiennent à une même sphère de la congruence). On peut montrer que la congruence de Ribaucour est caractérisée par la relation

$$\tilde{G}^{\alpha\beta}B_{i\alpha}\rho_\beta = \text{grad}$$

De ce fait et de (6) on obtient de nouveau le critère de la congruence de Ribaucour, précédemment obtenu par une autre voie [(2), formules (5)]. Mais maintenant, en s'appuyant sur la première des formules (2), on peut écrire ce critère en utilisant seulement les tenseurs de Bianchi (sans faire appel à ρ)

$$\frac{1}{(B, B)}B_{i\alpha}\nabla_\beta B^{\alpha\beta} = \text{grad}$$

ce qui est équivalent à l'affirmation suivante: une congruence de sphères de Ribaucour est caractérisée par le fait que dans la métrique des «distances tangentielles» $G_{\alpha\beta}du^\alpha du^\beta$, qui est le «tenseur de Tchébichev» (3) du réseau $B_{\alpha\beta}du^\alpha du^\beta = 0$, est un gradient.

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LITTÉRATURE CITÉE

[1] L. Bianchi, Lezioni di geometr. diff., II, cap. XIX, 1927. [2] J. Dubnov et M. Sabyrov, C. R. Acad. Sci. URSS, XXII, No. 8 (1939). [3] J. Doubnoff, C. R. 192, 261 et 1604 (1931).

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MATHEMATICS

ON THE PERIODICAL SOLUTIONS OF NON-LINEAR EQUATIONS OF THE FOURTH ORDER

By DSH. H. KARIMOV

(Communicated by S. L. Soboleff, Member of the Academy, 9. IX. 1945)

This paper is an outgrowth of an earlier investigation [1]. There, considering the equation

$$\frac{\partial^2 Z}{\partial t^2}+\frac{(2k+1)^2}{\pi^2}\frac{\partial^4 Z}{\partial x^4}=\Phi(x,t)+\mu f(Z) \tag{1}$$

subject to the conditions

$$\left.\begin{aligned} Z(0,t)&=\left.\frac{\partial^2 Z}{\partial x^2}\right|_{x=0}=0; \quad Z(x,0)=Z(x,1)\\ Z(1,t)&=\left.\frac{\partial^2 Z}{\partial x^2}\right|_{x=1}=0; \quad \left.\frac{\partial Z}{\partial t}\right|_{t=0}=\left.\frac{\partial Z}{\partial t}\right|_{t=1}\end{aligned}\right\} \tag{2}$$

it was proved that, if certain restrictions are improved on μ, $\Phi(x,t)$ and $f(Z)$, the above equation has a unique solution.

In the present note we examine a more general equation

$$\frac{\partial^2 Z}{\partial t^2}+\frac{(2k+1)^2}{\pi^2}\frac{\partial^4 Z}{\partial x^4}=\Phi(x,t)+\mu f\left(Z;\frac{\partial^2 Z}{\partial x^2}\right) \tag{3}$$

in the domain

$$\overline{D}=D\begin{pmatrix}0\leqslant x\leqslant 1\\ 0\leqslant t\leqslant 1\end{pmatrix}$$

This equation has a solution satisfying the conditions (2). Let the function $\Phi(x,t)$ be continuous and satisfy the Lipshitz condition

$$|\Phi(x_1,t_1)-\Phi(x_2,t_2)|\leqslant A|x_1-x_2|+B|t_1-t_2|$$

Let the function $\Phi_1(x,t)=\partial\Phi/\partial x$ and $\Phi_2(x,t)=\partial\Phi/\partial t$ be continuous in the domain $\overline{D}$ and of bounded variation in t and in x when $0\leqslant x\leqslant 1$ and $0\leqslant t\leqslant 1$; let $\Phi(x,t)$ be developable into an absolutely and uniformly convergent Fourier series

$$\Phi(x,t)=\Phi(x,t+1)=\sum_{n=0}^{\infty}\Phi_{2n+1}(t)\sin(2n+1)\pi x$$

where

$$\Phi_{2n+1}(t+1)=\Phi_{2n+1}(t)=2\int_0^1\Phi(\sigma,t)\sin(2n+1)\pi\sigma\,d\sigma$$

These conditions, which must be satisfied by the function $\Phi(x, t)$, will in the sequel be called A-conditions.

Theorem. *The differential equation* (3) *admits of a unique solution continuous with its partial derivatives of the fourth order in the domain* $\overline{D}$, *satisfies the conditions* (2) *and is periodical in* t *with the period equal to one, if:*

1. *The function* $\Phi(x, t)$ *satisfies the* A *conditions.*

2. $\left| f'\left(Z_1; \frac{\partial^2 Z_1}{\partial x^2}\right) - f'\left(Z_2; \frac{\partial^2 Z_2}{\partial x^2}\right)\right| \leqslant M_1 |Z_1 - Z_2| + M_2 \left|\frac{\partial^2 Z_1}{\partial x^2} - \frac{\partial^2 Z_2}{\partial x^2}\right|$.

3. $f(0, 0) = 0, \quad f\left(-Z; -\frac{\partial^2 Z}{\partial x^2}\right) = -f\left(Z; \frac{\partial^2 Z}{\partial x^2}\right)$.

4. $|\mu| < \frac{|a|\pi^2}{AN}$, *where* $a = \frac{2k+1}{\pi}$; $N = \sup |f'|$.

$$A = \sup \left[2 \int_0^t d\sigma \int_0^1 \left| \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi x \sin(2n+1)\pi\xi \sin(2n+1)^2 \pi^2 a (t-\sigma)}{(2n+1)^2} \right| d\xi + \right.$$

$$\left. + \int_0^t d\sigma \int_0^1 \left| \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi x \sin(2n+1)\pi\xi \cos(2n+1)^2 \pi^2 \left(t-\sigma+\frac{1}{2}\right)}{(2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}} \right| d\xi \right] \equiv$$

$$\equiv \sup \left[\int_0^t d\sigma \int_0^1 |K_1(x, t; \xi, \sigma)| d\xi + \int_0^t d\sigma \int_0^1 |K_2(x, t; \xi, \sigma)| d\xi \right]$$

We are going to solve this problem by the method of successive approximations.

For the zero approximation $Z_0(x, t)$ we take the solution of the equation

$$\frac{\partial^2 Z_0}{\partial t^2} + \frac{(2k+1)^2}{\pi^2} \frac{\partial^4 Z_0}{\partial x^4} = \Phi(x, t) \tag{4}$$

satisfying the conditions (2). The solution of this equation has the form

$$Z_0(x, t) = \sum_{n=0}^{\infty} S_{2n+1}^{(0)}(t) \sin(2n+1)\pi x \tag{5}$$

where

$$S_{2n+1}^{(0)}(t) = 2 \int_0^t d\sigma \int_0^1 \frac{\Phi(\xi, \sigma) \sin(2n+1)\pi\xi \sin(2n+1)^2 \pi^2 a (t-\sigma)}{a\pi^2 (2n+1)^2} d\xi +$$

$$+ \int_0^t d\sigma \int_0^1 \frac{\Phi(\xi, \sigma) \sin(2n+1)\pi\xi \cos(2n+1)^2 \pi^2 a \left(t-\sigma+\frac{1}{2}\right)}{a\pi^2 (2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}} d\xi$$

It is easily verified that $Z_0(x, t)$ satisfies the conditions (2) and is a function continuous with its fourth derivatives in the domain $\overline{D}$ and periodical in t with the period equal to one.

Having found the $(k-1)$-st approximation $Z_{k-1}(x, t)$, we find the k-th approximation $Z_k(x, t)$ as the solution of the equation

$$\frac{\partial^2 Z_k}{\partial t^2} + \frac{(2k+1)^2}{\pi^2} \frac{\partial^4 Z_k}{\partial x^4} = \Phi(x, t) + \mu f\left(Z_{k-1}; \frac{\partial^2 Z_{k-1}}{\partial x^2}\right) \tag{6}$$

under the conditions (2). This approximation has the form

$$Z_k(x, t) = \sum_{n=0}^{\infty} S_{2n+1}^{(k)}(t) \sin(2n+1)\pi x \tag{7}$$

where

$$S_{2n+1}^{(k)}(t) = \frac{2}{a\pi^2(2n+1)^2} \int_0^t d\sigma \int_0^1 \left[\Phi(\xi, \sigma) + \mu f\left(Z_{k-1}; \frac{\partial^2 Z_{k-1}}{\partial x^2}\right) \right] \times$$

$$\times \sin(2n+1)\pi\xi \sin(2n+1)^2 \pi^2 a (t-\sigma)\, d\xi +$$

$$+ \frac{1}{a\pi^2(2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}} \int_0^1 d\sigma \int_0^1 \left[\Phi(\xi, \sigma) + \mu f\left(Z_{k-1}; \frac{\partial^2 Z_{k-1}}{\partial x^2}\right) \right] \times$$

$$\times \sin(2n+1)\pi\xi \cos(2n+1)^2 \pi^2 a \left(t-\sigma+\frac{1}{2}\right) d\xi$$

There exists a limit of $Z_k(x, t)$ when $k \to \infty$

$$\lim_{k\to\infty} Z_k(x, t) = Z(x, t)$$

and this limit represents the solution of our problem.

To prove this it is sufficient to investigate the series

$$Z_0 + (Z_1 - Z_0) + \ldots + (Z_k - Z_{k-1}) + \ldots \tag{8}$$

It is easily seen that the numerical series

$$L \sum_{n=0}^{\infty} \left[\left| \frac{N}{a} \right| \left| \frac{MA}{\pi^2} \right|^n \right]$$

majorizes the series (8) and is convergent for sufficiently small values of μ. Consequently, the series (8) converges absolutely and uniformly in the domain $\overline{D}$. We may therefore write

$$Z(x, t) = \sum_{n=0}^{\infty} \frac{2}{a\pi^2(2n+1)^2} \int_0^t d\sigma \int_0^1 \left[\Phi(\xi, \sigma) + \mu f\left(Z; \frac{\partial^2 Z}{\partial x^2}\right) \right] \times$$

$$\times \sin(2n+1)\pi x \sin(2n+1)\pi\xi \sin(2n+1)^2 \pi^2 a (t-\sigma)\, d\xi +$$

$$+ \sum_{n=0}^{\infty} \frac{1}{a\pi^2(2n+1)^2 \sin\frac{(2n+1)^2\pi^2 a}{2}} \int_0^1 d\sigma \int_0^1 \left[\Phi(\xi, \sigma) + \mu f\left(Z; \frac{\partial^2 Z}{\partial x^2}\right) \right] \times$$

$$\times \sin(2n+1)\pi x \sin(2n+1)\pi\xi \cos(2n+1)^2 \pi^2 a \left(t-\sigma+\frac{1}{2}\right) d\xi \tag{9}$$

From the expression (9) we see that $Z(x, t)$ is a function of bounded variation in its first and second argument. Having noticed this, it is easy to prove the absolute and uniform convergence of the series

$$\sum_{k=0}^{\infty} \left[\frac{\partial Z_k}{\partial x} - \frac{\partial Z_{k-1}}{\partial x} \right] \qquad (Z_{-1} \equiv 0) \tag{10}$$

in the domain $\overline{D}$.

It is further easy to show the absolute and uniform convergence of the series

$$\sum_{k=0}^{\infty}\left[\frac{\partial^4 Z_k}{\partial x^4}-\frac{\partial^4 Z_{k-1}}{\partial x^4}\right] \quad \text{and} \quad \sum_{k=0}^{\infty}\left[\frac{\partial^2 Z_k}{\partial t^2}-\frac{\partial^2 Z_{k-1}}{\partial t^2}\right] \tag{11}$$

in the domain $\overline{D}$.

Thus, the obtained function $Z(x, t)=\lim_{k\to\infty} Z_k(x, t)$ is a continuous solution of the equation (3) in the domain $\overline{D}$, satisfying, with its derivatives entering into this equation, the conditions (2) and periodical in t with the period equal to one.

Let us now show the uniqueness of this solution. Suppose that there exist two different and continuous solutions Z and U of the equation (1) satisfying the conditions (2). Then the difference $Z-U=V$ is a solution of the equation

$$\frac{\partial^2 V}{\partial t^2}+a^2\frac{\partial^4 V}{\partial x^4}=\mu\left[f\left(Z;\frac{\partial^2 Z}{\partial x^2}\right)-f\left(U;\frac{\partial^2 U}{\partial x^2}\right)\right]$$

This difference is also the solution of the equation

$$V=Z-U=\frac{\mu}{a\pi^2}\int_0^t d\sigma\int_0^1\left[f\left(Z;\frac{\partial^2 Z}{\partial x^2}\right)-f\left(U;\frac{\partial^2 U}{\partial x^2}\right)\right]K_1(x, t;\ \xi, \sigma)\,d\xi+$$

$$+\frac{\mu}{a\pi^2}\int_0^1 d\sigma\int_0^1\left[f\left(Z;\frac{\partial^2 Z}{\partial x^2}\right)-f\left(U;\frac{\partial^2 U}{\partial x^2}\right)\right]K_2(x, t;\ \xi, \sigma)\,d\xi$$

whence

$$\sup|Z-U|\leqslant\frac{1}{\pi^2}\left|\frac{\mu}{a}\right|NA\sup|Z-U|$$

or

$$|\mu|\geqslant\frac{|a|\,\pi^2}{AN}$$

which contradicts the condition of the theorem.

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REFERENCES

[1] P. V. Soloviev, C. R. Acad. Sci. URSS, XXV, No. 9 (1939).

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MATHÉMATIQUES

SUR LES TRANSFORMATIONS OUVERTES DES ENSEMBLES *A*

Par L. KELDYCH

(Présenté par A. N. Kolmogoroff, de l'Académie, le 25. X. 1945)

Nous étudions dans cette Note les transformations ouvertes des ensembles A, situés dans l'espace de Baire I_x*. Soit φ une transformation continue de l'ensemble E en un ensemble E_1. On dit que φ est une transformation ouverte, si quel que soit l'ensemble UE ouvert dans E l'ensemble $\varphi(UE)$ est ouvert dans E_1. M. Hausdorff a démontré que l'image ouverte d'un ensemble du type G_δ est encore un ensemble du type G_δ (2). Nous démontrons ici qu'un théorème analogue n'est plus vrai pour les ensembles mesurables B de classes supérieures. Ensuite nous considérons les cas, dans lesquels un ensemble A peut être transformé en un autre ensemble A par une transformation ouverte.

Théorème 1. *Chaque ensemble A situé dans l'espace de Baire peut être obtenu au moyen d'une transformation ouverte à partir d'un ensemble du type* $G_\delta \cdot F_\sigma$.

L'ensemble M considéré du type $G_\delta \cdot F_\sigma$ doit être rigoureusement $G_\delta \cdot F_\sigma$ dans chaque intervalle contenant des points de M**. Si l'on pose $F_\sigma = \sum_{n=1}^{\infty} F_n$, les F_n étant fermés, $F_i \cdot F_j = 0$ si $i \neq j$, on a

$$M = G_\delta \sum_{n=1}^{\infty} F_n = \sum_{n=1}^{\infty} g_{\delta n}; \quad g_{\delta n} = G_\delta = F_n$$

Donc, M est la somme d'une infinité dénombrable d'ensembles $g_{\delta n}$ du type G_δ, telle que deux ensembles quelconques $g_{\delta i}$ et $g_{\delta j}$, $i \neq j$, sont séparables au moyen d'ensembles fermés. Il est naturel de poser la question: E étant un ensemble A du type $G_\delta \cdot F_\sigma$ au moins, peut on obtenir un ensemble A quelconque à partir de E au moyen d'une transformation ouverte? Nous démontrons que dans le cas général la réponse à cette question est négative. Mais nous indiquons certains cas, où une telle transformation est possible.

Théorème 2. *Soit E_1 un ensemble A quelconque situé dans l'espace de Baire, et E un ensemble A du type $G_\delta \cdot F_\sigma$ au moins et de première catégorie en lui-meme; alors on peut obtenir E_1 à partir de E au moyen d'une transformation ouverte.*

La condition d'être de première catégorie en lui-même est ici essentielle, car l'image ouverte d'un ensemble de seconde catégorie en lui-même est encore un ensemble de seconde catégorie en lui-même. Cette proposition suit faci-

* D'après M. Lusin, l'espace I_x peut être interprété comme l'ensemble de tous les points irrationnels du segment (0,1) de l'axe OX(1).

** C'est-à-dire M n'est dans aucun intervalle ni G_δ ni F_σ.

lement du théorème suivant dû à M. Mazurkiewicz (3): «Soient R, T deux espaces complets, séparables, $A \subset R$, f une transformation ouverte de A, $f(A) = B \subset T$. Il existe dans R un $G_\delta = A_1$ contenant A, et une transformation ouverte f_1 de A_1 telle que $f_1(A_1) \subset T$ et $f_1(x) = f(x)$ pour $x \subset A$.

Théorème 3. *Si un ensemble E, situé dans I_x, contient un G_δ partout dense sur cet ensemble, il en est de même pour son image ouverte $\varphi(E)$.*

Nous démontrons ce théorème sans rien supposer sur la nature de l'ensemble E. Mais si E est un ensemble A, ce théorème résulte du corollaire indiqué du théorème de M. Mazurkiewicz, car chaque ensemble A de seconde catégorie en lui-même contient un G_δ partout dense sur cet ensemble. Donc, si E est un ensemble A de seconde catégorie, on ne peut pas obtenir un ensemble A quelconque par une transformation ouverte de E. La question se pose alors: peut on dans ce cas obtenir au moyen d'une transformation intérieure de E un ensemble A quelconque de seconde catégorie? Mais il suit facilement des théorèmes de Mrs. Mazurkiewicz et Hausdorff que cette question est étroitement liée avec le problème de la puissance des ensembles CA (complémentaires aux ensembles A). En effet: si E est un ensemble A, qui peut par une transformation ouverte être transformé en un ensemble A de seconde catégorie et d'ailleurs quelconque, alors l'ensemble CE, qui est le complémentaire de E, contient un ensemble parfait.

Pour le voir remarquons que si $M = f(G_\delta)$ est une transformation continue, si pour $E \subset G_\delta$ on a $E_1 = f(E)$, où $M - E_1$ contient un ensemble parfait, alors $G_\delta - E$ contient un ensemble parfait. Soit maintenant E un ensemble A, et E_1 un ensemble A tel que, quel que soit un G_δ contenant E_1, l'ensemble $G_\delta - E_1$ contient un ensemble parfait*. Soit $E_1 = \varphi(E)$ une transformation ouverte. En vertu du théorème de M. Mazurkiewicz, il existe un $G_{\delta 0}$ contenant E et une transformation intérieure $\varphi_1(G_{\delta 0})$ coïncidant avec φ aux points de E. En vertu du théorème de M. Hausdorff $\varphi_1(G_{\delta 0}) = G_{\delta 1}$ est encore un ensemble du type G_δ. On a évidemment $E_1 \subset G_{\delta 1}$, et l'ensemble $G_{\delta 1} - E_1$ contient un ensemble parfait P. Donc, $\varphi_1^{-}(P)$ contient un ensemble parfait contenu dans $G_{\delta 0} - E$ et, par suite, dans CE.

Il en résulte que, s'il existe un ensemble A tel que son complémentaire ne contient aucun ensemble parfait, un tel ensemble ne peut pas être transformé en un ensemble A quelconque de seconde catégorie par une transformation ouverte.

Mais on peut montrer une classe d'ensembles A de seconde catégorie, qui peuvent être transformés par une transformation ouverte en un ensemble A de seconde catégorie et d'ailleurs quelconque.

Théorème 4. *Soit E un ensemble A de seconde catégorie tel qu'il existe une somme dénombrable dense sur E d'ensembles $P_i E$ parfaits dans E, chaque $P_i E$ étant au moins un $G_\delta \cdot F_\sigma$; alors on peut transformer E par une transformation ouverte en un ensemble A de seconde catégorie quelconque.*

Corollaire. Chaque ensemble A de seconde catégorie, dont chaque portion** est universelle***, peut être transformé en un ensemble A de seconde catégorie et d'ailleurs quelconque par une transformation ouverte.

Il serait intéressant de savoir: si E est un ensemble mesurable B de seconde catégorie et du type $G_\delta \cdot F_\sigma$ au moins, peut on transformer E en un ensemble A quelconque de seconde catégorie par une transformation ouverte?

Nous démontrons, enfin

Théorème 5. *Chaque ensemble A situé dans I_x dont aucune portion n'est pas un F_δ, peut etre transformé en l'espace de Baire par une transformation ouverte.*

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* E_1 peut, par exemple, être mesurable B non G_δ.

** On appelle portion d'un ensemble E chaque partie de E située dans un intervalle contenant au moins un point de E.

*** Un ensemble E situé dans I_2 est un ensemble A universel, si e étant un ensemble A quelconque, il existe un ensemble pE parfait dans E, qui est homéomorphe à e.

Pour établir les théorèmes 1, 2, 4 et 5 nous utilisons le même principe pour construire une transformation continue. Nous définissons les ensembles A au moyen des A-systèmes formés d'intervalles de Baire*. E et E_1 étant deux ensembles A, définis au moyen des A-systèmes correspondants C_E et C_{E_1}, pour construire une transformation continue $E_1 = \varphi(E)$ nous établissons une correspondance entre les éléments de C_E et C_{E_1}.

Soit $C = \{\Delta_n\}$ un A-système. Si un intervalle Δ_n est contenu dans Δ_m et s'il n'existe aucun intervalle de C contenu dans Δ_n et subordonné à Δ_m, nous dirons que Δ_n est neutre par rapport à Δ_m.

Voici maintenant le principe utilisé.

Lemme. Soient $C_E = \{\Delta_n\}$ et $C_{E_1} = \{\delta_m\}$ deux A-systèmes définissant respectivement les ensembles E et E_1. Entre les éléments de ces A-systèmes on établit une correspondance de telle manière qu'à chaque élément Δ_n corresponde un seul élément δ_m, et à chaque élément δ_m corresponde un nombre fini d'éléments de C_E. Cette correspondance vérifie les conditions suivantes:

1) Si Δ_i correspond à δ_i et Δ_j correspond à δ_j, et si Δ_i est subordonné à Δ_j, alors δ_i est subordonné à $\delta_{j'}$.

2) Parmi les éléments de C_E correspondant à un élément δ_m de rang r il existe au moins un élément de rang r.

3) Si Δ_i correspond à δ_i, Δ_j correspond à $\delta_{j'}$ et $\Delta_i \subset \Delta_j$, on a en général $\delta_{i'} \subset \delta_{j'}$; mais il peut exister pour Δ_j un nombre fini d'intervalles $\Delta_{i_1}, \Delta_{i_2}, \ldots, \Delta_{i_s}$ neutres par rapport à Δ_j tels que les intervalles qui leur correspondent $\delta_{i'_1}, \delta_{i'_2}, \ldots, \delta_{i'_s}$ et les intervalles contenus dans tous les δ_i ne sont pas contenus dans $\delta_{i'}$.

Alors la correspondance établie entre les éléments de C_E et de C_{E_1} définit une transformation continue $E_1 = \varphi(E)$ de E en E_1.

Pour que la transformation $\varphi(E)$ soit ouverte, on doit remplacer la condition 2) par la condition suivante:

2') Soit Δ_{i_s} un élément quelconque de C_{E_1} correspondant à δ_i; alors à chaque élément δ_j contenu dans δ_i correspond au moins un élément Δ_{j_s} contenu dans Δ_{i_s}, et à chaque élément δ_l subordonné à δ_j correspond au moins un élément Δ_{l_s} subordonné à Δ_{i_s}.

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LITTÉRATURE CITÉE

[1] N. Lusin, Leçons sur les ensembles analytiques, Paris, 1930. [2] F. Hausdorff, Fundamenta Math., XXIII, 279 (1934). [3] Mazurkiewicz, Fundamenta Math., XIX, 198 (1932).

* Les points de I_x étant représentés au moyen de fractions continues infinies, un intervalle de Baire de rang n $\delta_{a_1 a_2 \ldots a_n}$, est l'ensemble de tous les points de I_x ayant les n premiers indices fixes et respectivement égaux à $a_1, a_2, \ldots, a_n$.

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MATHEMATICS

ON PAIRS OF CONJUGATE PARALLEL TRANSLATIONS IN n-DIMENSIONAL SPACES

By A. NORDEN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 10. IX. 1945)

A symmetric non-degenerate tensor b_{ij} on the n-dimensional space of symmetric affine connectivity A_n will be called pseudo-metric if

$$\nabla_{[k}b_{j]l} = \omega_{[k}b_{j]l} \tag{1}$$

or

$$\nabla_k b_{ij} = \omega_k b_{ij} + b_{ijk} \tag{2}$$

where b_{ijk} is a symmetric tensor.

If we put

$$\bar{\omega}_k = \omega_k + d_k \lg \lambda; \quad \bar{b}_{ijk} = \lambda b_{ijk} \tag{3}$$

the equations (1) and (2) remain valid under the substitution $b_{ij} = \lambda b_{ij}$.

Fixing b_{ij} is equivalent to fixing a cone of the second order which contains the directions satisfying the condition

$$b_{ij}v^i v^j = 0 \tag{4}$$

or the asymptotic directions of the pseudo-metric.

The hyperplane

$$w_i = b_{ia}v^a \tag{5}$$

polar to the direction v^i with respect to the cone (4) will be called conjugate to this direction. If the direction v^i can be translated parallel, together with its conjugate plane, it will be called the principal direction corresponding to the given direction of translation du^i. The principal directions are determined by the system

$$(b_{ij}du^k - sb_{ij})\,v^j = 0 \tag{6}$$

which is consistent, provided s is a root of the characteristic equation

$$\mathrm{Det}\,|\,b_{ijk}du^k - sb_{ij}\,| = 0 \tag{7}$$

The condition (1) is equivalent to the existence of such a cone of reciprocal directions which, being considered in pairs, satisfy the condition

$$b_{ijk}v^j\omega^k = 0 \tag{8}$$

Hence follows that every direction of this mark is the principal direction with respect to any other considered as a direction of parallel translation. The cone of reciprocal directions has the equation

$$\mathrm{Det}\,|\,b_{ijk}v^k\,| = 0 \tag{9}$$

We shall say that two affine connectivities G and Γ with symmetric coefficients g^k_{ij} and Γ^k_{ij} are conjugate to each other with respect to a given pseudo-metric, if with every direction that can be translated parallel to itself in one of them, say, in G, a conjugate hyperplane is associated which can be translated parallel to itself in Γ.

The conditions of conjugateness of translations have the form

$$d_k b_{ij} - g^s_{ik} b_{sj} - \Gamma^s_{jk} b_{is} = d_k b_{ij} - \Gamma^s_{ik} b_{sj} - g^s_{jk} b_{is} = \omega_k b_{ij} \tag{10}$$

Given one of the translations and the pseudo-metric, the other translation is then determined uniquely, their coefficients being subject to the relation

$$\Gamma^k_{ij} - g^k_{ij} = b^{ks} b_{sij} \tag{11}$$

where b^{ks} is the tensor reciprocal to b_{ij}. The coefficients

$$z^k_{ij} = \frac{1}{2}(g^k_{ij} + \Gamma^k_{ij}) \tag{12}$$

define the mean metric of the given pair, which is Weyl's metric with the fundamental tensor b_{ij} because

$$\partial_k b_{ij} - z^s_{jk} b_{is} - z^s_{jk} b_{is} = \omega^s_k b_{ij} \tag{13}$$

For the curvature tensors of the conjugate connectivities the following relation takes place

$$R^s_{ijp} b_{sq} + \rho^s_{ijq} b_{ps} = \omega_{[ij]} b_{pq} \tag{14}$$

Let us note some particular cases.

1. Weyl's pair. If one of the conjugate connectivities is Weyl's connectivity with the fundamental tensor g_{ij}, the other connectivity is also Weyl's connectivity with the fundamental tensor

$$\gamma_{ij} = \lambda b_{ir} b_{js} \tilde{g}^{rs} \tag{15}$$

where $\tilde{g}^{ij}$ is the tensor reciprocal to g_{ij}. The cones of absolute directions of conjugate Weyl's metric are mutually polar with respect to the asymptotic cone of the pseudo-metric. The angle between any two directions equals that formed by their conjugate hyperplanes when measured in different metrics.

2. Conform pair. If

$$\gamma_{ij} = \mu g_{ij} \tag{16}$$

then the cones of absolute directions are autopolar with respect to the asymptotic cone and coincide, and the two conjugate metrics are conform.

3. Self-conjugate pair. If

$$b_{ij} = \nu g_{ij} \tag{17}$$

then $b_{ijk} = 0$, and the conjugate connectivities coincide. Thus, every Weyl's metric is self-conjugate with respect to the cone of its absolute directions.

4. Codazzi's pair. The mean metric of a pair is the Riemann metric if and only if the vector ω_k is the gradient. In this case the tensor b_{ij} can be so normalized as to have $\omega_k = 0$. The tensor b_{ij} will then be found to satisfy Codazzi's equation

$$\nabla_{[kb]_i} = 0 \tag{18}$$

with respect to both connectivities.

5. Equi-affine pair. The connectivity conjugate to an equi-affine connectivity is equi-affine if and only if the pair is Codazzi's pair. In general, the assertions: a) the connectivity G is equi-affine, b) the connectivity Γ is equi-affine, c) G and Γ form a Codazzi's pair, are all true whenever any two of them are true.

If W, Ω and Q denote the volumes of elementary domains in the connectivities g, Γ and the mean metric, respectively, then

$$\partial_i \lg\left(\frac{\Omega}{W}\right) = t_i = b^{rs} b_{rsi} \tag{19}$$

where t_i is the Chebyshev pseudo-metric tensor, and

$$Q = c\sqrt{WQ} \tag{20}$$

where c is constant. If the Chebychev tensor of an equi-affine tensor becomes zero, we shall have a constant ratio between the corresponding volumes in all three connectivities.

A particular form of equi-affine pair can be obtained by supposing g to be an equi-affine projective Euclidean connectivity, because in this case the general solution of Codazzi's equation has the form

$$b_{ij} = \nabla_j \varphi_i - \frac{\varphi}{n-1} R_{ij} \tag{21}$$

where $\varphi_i = \partial_i \varphi$, $R_{ij} = R^{\kappa}_{\kappa ij}$, and φ is an arbitrary function. In particular, if $\varphi = \text{const}$, the conjugate connectivity is also projective Euclidean.

6. Riemann's pair. Equi-affine Weyl's pair consists of two conjugate Riemann geometrics. Their fundamental tensors are connected by the relation

$$\gamma_{ij} = b_{ir} b_{js} \tilde{g}^{rs} \tag{22}$$

where b_{ij} is a tensor satisfying Codazzi's equation. A particular form of Riemann's pair can be obtained by taking into account that in every conform-Euclidean Riemann geometry with the fundamental metric tensor g_{ij} the tensor

$$L_{ij} = -R_{ij} + \frac{1}{2(n-1)} g^{\alpha\beta} R_{\alpha\beta} g_{ij} \tag{23}$$

satisfies Codazzi's equation (1). Hence follows that if a pseudo-metric is introduced by means of the tensor

$$b_{ij} = L_{ij} + c g_{ij} \quad (c = \text{const}) \tag{24}$$

then the conjugate connectivity is also Riemann's connectivity. In particular, if $c = 0$, the conjugate connectivity is also conform-Euclidean.

Another particular case of Riemann's pair can be obtained by defining a pseudo-metric by the formula (21) in a space of constant curvature.

7. Euclidean pair. The equation (14) implies that for $n > 2$ Codazzi's tensors only define pseudo-metrics in Euclidean space, and the connectivity conjugate to an Euclidean connectivity is Euclidean itself. According to (21), the general solution of Codazzi's equation in Euclidean space is

$$b_{ij} = \nabla_j \varphi_i \tag{25}$$

where φ is an arbitrary point function. By subjecting φ to the condition $\operatorname{Det} |\Delta_j \varphi_i| \neq 0$, we can obtain any Euclidean pair in this way.

For $n = 2$ conjugate pairs possess a quantity of special properties. Every non-degenerate tensor b_{ij} is pseudo-metric in this case. To give such a tensor is tantamount to giving a line net, while the condition of conjugateness makes to every direction, which can be translated parallel in the connectivity G, correspond a direction divided with it harmonically by the directions of the net and admitting parallel translation in the conjugate connectivity Γ. Thus, in case $n = 2$, every affine connectivity can be completed to a conjugate pair by means of a pseudo-metric defined by an arbitrary net. In particular, the connectivity conjugate to an Euclidean connectivity is, in general, quasi-Euclidean, that is, it admits of absolute parallelism of directions, though its curvature is not necessarily zero.

A conform pair of Weyl's connectivities for $n = 2$ is either self-conjugate or characterized by orthogonality of the net defining the pseudo-metric. Then and only

then can there exist ∞^1 nets with respect to which the connectivities of the pair are conjugate. The tensor b_{ij} of a conform pair of Euclidean connectivities is determined by (25) (provided $n=2$) if φ is a harmonic function. The theory of equi-affine pairs was expounded in detail by the author(2). The fact that the tensor (19) vanishes characterizes the Chebychev net.

The domain of the theory of conjugate pairs is determined by the following consideration. On every normalized hypersurface of the projective plane a pair of affine connectivities can be defined that are conjugate with respect to the cone of asymptotic directions(3). By changing one of the normalizing manifolds one of the connectivities undergoes a projective transformation, *i. e.* a transformation which leaves invariant its geodesics. By changing the two normalizing manifolds simultaneously the coefficients of the connectivities are being transformed as follows

$$\left.\begin{aligned} \bar{g}^k_{ij} &= g^k_{ij} + \delta^k_i p_j + \delta^k_j p_i - b_{ij} b^{km} \bar{n}_m \\ \bar{\Gamma}^k_{ij} &= \Gamma^k_{ij} + \delta^k_i \pi_j + \delta^k_j \pi_j - b_{ij} b^{km} p_m \end{aligned}\right\} \qquad (26)$$

where p_i and π_i are vectors of the projective transformations indicated above. On every hypersurface an Euclidean pair of connectivity can be defined by assuming the normals of the 1st kind to intersect at one point which is incident with the hyperplane containing all the normalizing manifolds of the 2nd kind. Hence follows that a conjugate pair of connectivities is realizable on a normalized hypersurface if and only if it can be carried into an Euclidean pair by the transformation (26).

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REFERENCES

[1] Рашевский, Введение в риманову геометрию, 1936, стр. 164. [2] А. Норден, Тр. семинара по векторн. и тензорн. анализу, в. IV, 1937. [3] A. Norden, C. R. Acad. Sci. URSS, XLVIII, No. 8 (1945).

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MECHANICS

ON FUEL CONSUMPTION BY A ROCKET CROSSING THE ATMOSPHERE WITH CONSTANT ACCELERATION

By A. STERNFELD

(Communicated by L. S. Leibenson, Member of the Academy, 25. VIII. 1945)

The computation method previously developed by the author for certain particular cases of rockets crossing the atmosphere [(1), p. 193—206] is generalized in the present paper, whose purpose it is to show that the amount of fuel required to overcome the resistance of the air to the flight of a rocket moving with constant acceleration away from the Earth surface, is defined by the following formula:

$$m = \frac{C\,S\Gamma}{\delta \sin\beta} K\,(182.844 + 79.580\,K + 21.665K^2 + 4.618K^3 + 0.843K^4 + 0.138K^5) \quad (1)$$

Here β is the angle of start of the rocket with reference to the horizon; Γ is the acceleration of its motion in m/sec.^{2}; γ is the perceivable acceleration due to the reactive force, *i. e.* that which would be recorded by an acceleration indicator placed within the rocket, in m/sec.^{2}; c is the velocity of the outflowing gases relative to the rocket (jet velocity), in km/sec.^{2}; C_x is the aerodynamic coefficient of resistance; S is the maximum cross-section of the rocket perpendicular to the direction of flight in m^2; m is the mass of the fuel in kg, and

$$K = \frac{0.1\gamma}{c\sqrt{0.1\Gamma \sin\beta}} \quad (2)$$

As, indeed, we have $v = \sqrt{2\Gamma s}$ for the velocity of a rocket moving with constant acceleration (s is the distance travelled by the rocket), the Galileo-Newton formula for air resistance takes the form

$$R = C_x \delta s S \Gamma \quad (3)$$

where δ is the density of the air.

The mass of the fuel consumed in overcoming the resistance of the air during an interval Δt when the jet velocity is c, will be

$$\Delta m_0 = \frac{R}{c}\,\Delta t \quad (4)$$

The acceleration of the rocket would be reduced by the added weight of the «main» store of fuel m_0 necessary to overcome the resistance of the air. To avoid this reduction in acceleration an additional amount of fuel m_1 is required in order to overcome the inertia of the «main» mass m_0. Again, for overcoming the inertia of the mass m_1 one has to provide the rocket with another mass of fuel m_2, and so on.

The added consumption of fuel during the interval Δt is

$$\Delta m_1 = \frac{m_0 \gamma}{c} \Delta t \qquad (5)$$

$$\Delta m_2 = \frac{m_1 \gamma}{c} \Delta t \qquad (6)$$

where m_0 and m_1 denote the instantaneous mass of fuel left in the rocket at a given instant, and are determined by integrating equations (4) and (5).

Composing an unlimited number of equations similar to (4), (5), (6) and so on, integrating them and taking their sum we shall obtain the answer to the proposed problem. For practical purposes it will suffice to take into account the first, second and third supplementary masses of fuel. But in the results of the calculations given above we have also considered the fourth and fifth. It was assumed that $\Gamma' = \gamma' = 10$ m/sec.^{2}, $c' = 1$ km/sec., $S' = 1$ m^2, $C_x' = 0.1$. The density of the air was supposed to conform to the laws of the international standard atmosphere.

Making use of the similarity laws, the results of numerical integration obtained for a particular case can be extended to any case of rocket motion with constant acceleration.

In fact, we may write the following relation for the instantaneous mass of the «main» store of fuel

$$\frac{m_0}{m_0'} = \frac{\Delta m_0}{\Delta m_0'} = \frac{R t'}{R' c} \frac{\Delta t}{\Delta t'}$$

Now, it follows from (3) that at a given height

$$\frac{R}{R'} = \frac{C_x}{C_x'} \frac{S}{S'} \frac{\Gamma}{\Gamma'} \frac{\sin \beta'}{\sin \beta} \qquad (7)$$

since $s = h/\sin \beta$, and

$$\frac{\Delta t}{\Delta t'} = \frac{t}{t'} = \sqrt{\frac{\Gamma'}{\Gamma} \frac{\sin \beta'}{\sin \beta}} \qquad (8)$$

Therefore

$$\frac{m_0}{m_0'} = \frac{C_x}{C_x'} \frac{S}{S'} \left(\frac{c}{c'}\right)^{-1} \left(\frac{\Gamma}{\Gamma'}\right)^{1/2} \left(\frac{\sin \beta}{\sin \beta'}\right)^{-3/2} \qquad (9)$$

Setting

$$\left(\frac{c}{c'}\right)^{-1} \frac{\gamma}{\gamma'} \left(\frac{\Gamma}{\Gamma'}\right)^{1/2} \left(\frac{\sin \beta}{\sin \beta'}\right)^{-1/2} = K \qquad (10)$$

it is easy to find that for the first supplementary consumption of fuel

$$\frac{m_1}{m_1'} = \frac{\Delta m_1}{\Delta m_1'} = \frac{m_0}{m_0'} K \qquad (11)$$

and for the n-th consumption

$$\frac{m_n}{m_n'} = \frac{\Delta m_n}{\Delta m_n'} = \frac{m_0}{m_0'} K^n \qquad (12)$$

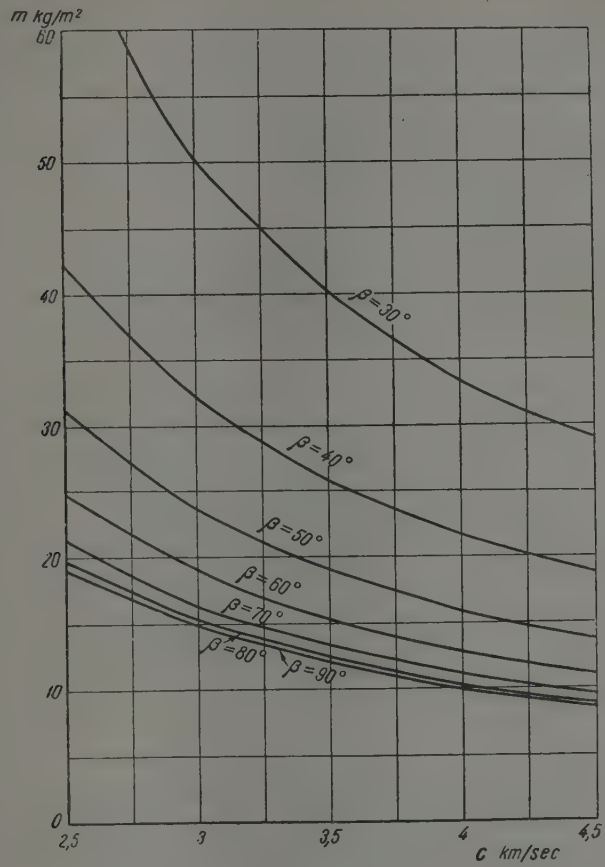
Thus, the similarity law for the total amount of fuel necessary to cross the atmosphere is expressed by the formula

$$m = \frac{C_x}{C_x'} \frac{S}{S'} \frac{\gamma'}{\gamma} \frac{\Gamma}{\Gamma'} \frac{\sin \beta'}{\sin \beta} K (m_0' + m_1' K + m_2' K^2 + m_3' K^3 + \ldots) \qquad (13)$$

Substituting in this equation the respective values for the vertical start of the rocket ($\beta = 90$) found by numerical integration while using the above-mentioned values for the symbols in equation (13) ([2]), we arrive at the proposed formula (1).

By means of equation (13) and the data given in ([2]) the problem we are interested in can as rapidly be solved for any instant of the flight of the

rocket through the atmosphere. This is particularly useful in the case of a step-rocket or a rocket sent from a height h above sea level with a velo-



city $v = \sqrt{2\Gamma h/\sin\beta}$. In the adjoined table are given some values of m'_0/C'_x, m'_1/C'_x, *etc.*, for $C'_x = 0.1$.

h, km	m'_0/C'_x	m'_1/C'_x	m'_2/C'_x	m'_3/C'_x	m'_4/C'_x	m'_5/C'_x
0	182.844	79.580	21.665	4.618	0.843	0.138
2	166.869	43.644	8.164	1.276	0.179	0.023
4	144.341	30.452	4.963	0.706	0.093	0.012
6	120.687	21.818	3.240	0.434	0.055	0.007
8	98.238	15.796	2.197	0.283	0.035	0.004
10	78.087	11.516	1.530	0.192	0.024	0.003
15	41.259	5.442	0.679	0.082	0.009	0.001
20	21.243	2.691	0.327	0.038	0.004	0.000
25	10.764	1.325	0.154	0.017	0.002	0.000
30	5.268	0.598	0.065	0.007	0.001	0.000
35	2.556	0.271	0.028	0.003	0.000	0.000
40	1.230	0.124	0.012	0.001	0.000	0.000
45	0.590	0.057	0.005	0.000	0.000	0.000
50	0.282	0.027	0.002	0.000	0.000	0.000
60	0.065	0.007	0.000	0.000	0.000	0.000
70	0.015	0.002	0.000	0.000	0.000	0.000

Making use of formula (1) and the table of values of Γ/γ([1], p. 182—184), the author has determined the consumption of fuel on the traversing of the atmosphere by rockets with $\gamma=4$ $g_0=39.26$ $m/\sec^2$, starting at an angle β ranging from 30 to 90° (see figure).

It should be noted that in the computations we are concerned with g, and thereby Γ, can be looked upon as constant and equal to their initial values. Verification shows that values obtained on this assumption are overestimated by some hundredths of one per cent only. As for the curvature of the Earth, it can be neglected for not very small angles of start, so that the trajectory of the rocket in the atmosphere may be regarded as a straight line.

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REFERENCES

[1] А. Штернфельд, Введение в космонавтику, 1937.

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THEORY OF ELASTICITY

ON THE FUNDAMENTAL PROBLEM IN THE THEORY OF MINING PRESSURE

By S. MICHLIN

(Communicated by V. I. Smirnov, Member of the Academy, 21. IV. 1945)

According to custom, the excavations to be discussed in this paper are supposed to extend over a sufficiently large distance in the horizontal direction. Thereby we are allowed to treat the stress problem as a plane one. The height of an excavation is taken to be small in comparison with its breadth, so that its cross-section presents an infinitely narrow horizontal slit. Finally, all of the excavations are supposed to be located at the same depth, and if infinite in number, they are assumed to be identical in size and position.

The OX-axis is defined to be directed along the line of excavation, and the OY-axis to run upward at right angles to it.

The stresses σ_x, σ_y, τ_{xy} may be represented as follows ([1],[2])

$$\sigma_x = \sigma'_x + \sigma''_x, \quad \sigma_y = \sigma'_y + \sigma''_y, \quad \tau_{xy} = \tau'_{xy} + \tau''_{xy} \tag{1}$$

where

$$\sigma'_x = \frac{\sigma}{1-\sigma} g\rho\,(y-f), \quad \sigma'_y = g\rho\,(y-f), \quad \tau''_{xy} = 0 \tag{2}$$

are the stresses in the massif (intact rock), and σ''_x, σ''_y, τ''_{xy} are the added stresses caused by the excavations. The stresses due to the excavations vanish at infinity. In formulae (2) σ is Poisson's ratio; ρ is the density of the massif, which for simplicity is supposed to be homogeneous; f is the depth of the excavations.

Proceeding on the considerations set forth in ([1]) and ([2]) we can formulate our problem as follows: To find the added stresses σ''_x, σ''_y, τ''_{xy} satisfying the equations of the theory of elasticity when there are no volume forces; these stresses are to be determined in a plane broken along the OX-axis by a number of cuts corresponding to the excavations. The added stresses vanish at infinity, and are subject at the cuts to the following boundary conditions

$$\tau''_{xy} = 0, \quad \sigma''_y = h \tag{3}$$

where $h = \rho f$ is the weight of a column of rock resting on the excavation and with cross-section area equal to unity.

If there is a finite number of excavations, the problem can be solved by the method described in an earlier paper ([3]).

Denote by C the totality of twice crossed cuts. As the cuts are lying on the real axis, we have $\zeta = \bar{\zeta}$, where ζ is the complex coordinate of the point C. Substitute now in formula (4.2) [([3]), p. 31] the factor ζ under the integral sign with $\bar{\zeta}$ and the factor ζ in formula (5.2) with ζ. Since $\varphi'(z)$

is single-valued, we shall find, using formulae (19.1) and (20.1) [(3), p. 19], that

$$\varphi(z) = -\frac{1}{\varkappa} F(z) + \text{const}, \quad \psi(z) = \Phi(z) - z\varphi'(z) + \text{const}$$

where $\varkappa = 1$, and $F(z)$ and $\Phi(z)$ are known functions defined by formulae (3.2). For computing them we must know the Schwarz kernel. In the case of a plane cut up along a finite number of segments of the real axis the Schwarz kernel has the following form

$$\frac{1}{4\pi} T(z; \zeta) = \frac{1}{\zeta - z}\left[1 - \sum_{k=1}^{n} \alpha_k(\zeta) \ln|\zeta + \sqrt{R_k(\zeta)}\,|\right]\left(1 + \sqrt{\frac{R(\zeta)}{R(z)}}\right) +$$

$$+ \sum_{k=0}^{n} \frac{P_k(z)\,\zeta^k}{\sqrt{R(z)}} + \sum_{k=1}^{n} \alpha_k(\zeta) \ln\left(z + \sqrt{R_k(z)}\right)$$

Here n is the number of cuts; a_k and b_k, the beginning and end of a k-th cut; $R_k(z) = (z - a_k)(z - b_k)$; $R(z) = \prod_{k=1}^{n} R_k(z)$.

The functions $\alpha_k(\zeta)$ and $P_k(\zeta)$ are determined from the relations

a) $\sum_{k=1}^{n} \alpha_k(\zeta) = 0$

b) for $z = a_k$ and $z = b_k$ $(k = 1, 2, \ldots, n)$

$$\frac{\sqrt{R(\zeta)}}{\zeta - z}\left(1 - \sum_{k=1}^{n} \alpha_k(\zeta) \ln|\zeta + \sqrt{R_k(\zeta)}\,|\right) + \sum_{k=0}^{n} P_k(\zeta)\, z^k = 0$$

However, the method developed in (2) leads to complicated quadratures, and the actual determination of the stresses proves to be a difficult matter. The solution is easier to obtain by making use of the results secured by Muskhelishvili (4). For the case where the number of excavations is infinite, the solution (though not an effective one) of the more general problem has been given by Savin (5). The present paper will show that the problem of stresses in rocks can be solved in an elementary manner both for a finite and infinite number of excavations.

As has been shown in (2), the second of the conditions (3) entails as a consequence that the stresses σ_x'', σ_y'', τ_{xy}'' can be expressed through a single function $\varphi(z)$, regular in the cut plane, by means of the formulae

$$\sigma_x'' + \sigma_y'' = 4\,\mathrm{Re}\,(\varphi'(z)) \tag{4}$$

$$\sigma_y'' - \sigma_x'' + 2i\tau_{xy}'' = -4iy\varphi''(z), \quad z = x + iy \tag{5}$$

From (5) it follows that on the OX-axis $\sigma_x'' = \sigma_y'' = 0$. Now we can deduce from (3) and (4) a boundary condition for the function $\varphi(z)$, viz.

$$\mathrm{Re}\,(\varphi'(z)) = h/2 \text{ on the cuts} \tag{6}$$

Moreover, $\varphi(z)$ is regular in the cut plane.

Let the number of excavations be finite and the plane cut up along the segments (a_k, b_k), $k = 1, 2, \ldots, n$, of the real axis. The function

$$\varphi'(z) = \frac{h}{2}\left(1 - \frac{z^n + A_1 z^{n-1} + \ldots + A_n}{\sqrt{\prod_{k=1}^{n}(z - a_k)(z - b_k)}}\right) \tag{7}$$

is regular in the cut plane and satisfies the condition (6) if the coefficients

$A_1, \ldots, A_n$ are arbitrary real numbers. Let us subject them to the equation

$$\sum_{k=0}^{n} A_k \int_{a_m}^{b_m} \frac{z^{n-k}\,dz}{\sqrt{\prod_{j=1}^{n}(z-a_j)(z-b_j)}} = 0; \quad A_0 = 1 \tag{8}$$

Then $\varphi(z)$ will be regular in the cut plane, and the formula (7) gives the solution of the problem.

The coefficients A_k can be expressed through elliptic integrals for $n=2$ and through hyperelliptic integrals for $n>2$. They can be easily computed if the dimensions and position of the excavations are known. After that the stresses are determined very simply from formulae (4) and (5). It may be remarked in addition that the fact of system (8) being solvable follows immediately from the uniqueness theorem in the plane problem of the theory of elasticity.

Let us now proceed to the case where the number of excavations is infinite, and they are all alike and similarly situated. Denote the distance from one excavation to the next, as measured between the middles of their cross-sections, by $2a$, and the breadth of a cross-section by $2b$. Define, as before, the OX-axis to coincide with the line along which the excavations are arranged, and the OY-axis to be drawn through the middle of one of the sections.

The function $\varphi'(z)$ is determined from the following conditions: 1) $\varphi'(z)$ is periodical and its strip of periods is $-a \leqslant x \leqslant a$; 2) the condition (6) is fulfilled on the $(-b, b)$ cut; 3) $\varphi'(z) \to 0$ as $y \to \pm\infty$; 4) $\varphi(z)$ is single-valued in the cut plane. We can use the function

$$\tau = \operatorname{tg}\frac{\pi z}{2a} \tag{9}$$

to map the strip of periods onto the τ-plane. The cut $(-b, b)$ in the strip of periods transforms into the cut $(-\gamma, \gamma)$ of the τ-plane, and we have $\gamma = \operatorname{tg}\frac{\pi b}{2a}$.

Put $\varphi'(z) = \Phi(\tau)$. The conditions 1)—4) are thereby transformed as follows: 1) $\Phi(\tau)$ is regular in the τ-plane, cut along the segment $(-\gamma, \gamma)$ of the real axis; 2) $\operatorname{Re}(\Phi(\tau)) = h/2$ on the $(-\gamma, \gamma)$ cut; 3) $\Phi(\pm i) = 0$; 4) if the $(-\gamma, \gamma)$ cut is comprised within the contour Γ, then

$$\int_{\Gamma} \frac{\Phi(\tau)}{1-\tau^2}\,d\tau = 0$$

All of these conditions are satisfied by the function

$$\Phi(\tau) = \frac{h}{2}\left(1 - \frac{\tau}{\sqrt{\tau^2-\gamma^2}\cos\frac{\pi b}{2a}}\right) \tag{10}$$

or, if we revert to the variable z,

$$\varphi'(z) = \frac{h}{2}\left(1 - \frac{\sin\frac{\pi z}{2a}}{\sqrt{\cos^2\frac{\pi b}{2a} - \cos^2\frac{\pi z}{2a}}}\right) \tag{11}$$

From (11) follows the immediate consequence that in the case of infinite number of excavations the mean pressure on the massif is $\frac{a}{a-b}h$.

If there is a finite number of excavations, the mean pressure may be considerably lower than the above value [1].

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21. IV. 1945.

REFERENCES

[1] Ц. О. Левина и С. Г. Михлин, Тр. Сейсмологич. ин-та АН СССР, № 94 (1940). [2] S. Michlin, Bull. Acad. Sci. URSS, sér. techn., № 7—8 (1942). [3] С. Г. Михлин, Тр. Сейсмолог. ин-та АН СССР, № 65 (1935). [4] Н. И. Мусхелишвили, Сообщ. АН ГрузССР, III, № 2 (1942). [5] G. N. Savin, C. R. Acad. Sci. USSR, XXIII, No. 6 (1939).

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MATHEMATICAL PHYSICS

ON POINCARÉ'S PROBLEM IN THE THEORY OF POTENTIAL

By D. I. SHERMAN

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§ 1. Suppose that in the plane of the complex variable $z=x+iy$ we are given a finite multiconnected domain S, bounded by $m+1$ closed curves $L_q\,(q=0, 1, \ldots, m)$ with continuous curvature having no points in common. By L_0 we shall denote the outer boundary of the domain, comprising all other, inner, boundaries $L_q\,(q=1, \ldots, m)$. The contour $L=L_0+L_1+\ldots+L_m$ is defined to be positive with respect to the domain S, and the normal n to it is supposed to be directed from the inside of S. Further we assume that the origin lies inside S. Finally, we denote by S_q the simply connected domains bounded by the corresponding curves L_q. Evidently, S_0 will be infinite. Let z_q denote certain arbitrarily fixed points situated inside the $S_q\,(q=1, 2, \ldots, m)$.

Poincaré's problem consists, as is known, in the determination of the function $u(x, y)$ harmonic in the domain S and satisfying on L the condition

$$a(s)\frac{\partial u}{\partial x}+b(s)\frac{\partial u}{\partial y}+c(s)u=f(s) \qquad (1)$$

where $a(s)$, $b(s)$, $c(s)$ and $f(s)$ are given functions of the arc length s. We shall suppose that $a(s)$ and $b(s)$ do not vanish simultaneously and satisfy Hölder's condition, and that $c(s)$ and $f(s)$ are continuous on L.

Assuming that $a^2+b^2=1$ and putting $\alpha+i\beta=e^{\int\frac{c\,ds}{a+ib}}$ we shall suppose that the function $\alpha(s)$ receives an increment equal to $2\pi\varkappa_q$ ($\varkappa_q$ integer) along the curves $L_q\,(q=0, 1, \ldots, p)$ transcribed in the indicated direction and equal to $-2\pi\varkappa_q$ ($\varkappa_q>0$) along the curves $L_q\,(q=p+1, \ldots, m)$.

It may be shown that the required harmonic (one-valued) function is representable in S in the form

$$u(x, y)=\frac{1}{\pi}\int_L \nu(s)\,\mathrm{Re}\left(\frac{1}{t-z}\right)ds+\mathrm{Re}\left\{\sum_{q=1}^{m} c_q \ln(z-z_q)+c_0-g(z)\right\} \qquad (2)$$

where Re denotes the real part, $t=\xi+i\eta$ is the affix of the point on L, $z=x+iy$

on L_0 and $\varepsilon = -1$ on L_q $(q = 1, 2, \ldots, m)$. Further

$$\left.\begin{aligned} c_{00} &= \frac{\alpha}{2\pi i}\int_{L_0} \nu(s)\frac{t^{n_0-1}}{a+ib}\,dt, \quad c_{q0} = \frac{i}{\pi}\int_{L_q} \nu(s)\,\mathrm{Re}\,\frac{dt}{a+ib} \\ & (q = 1, 2, \ldots, m) \\ g(z) &= \sum_{q=p+1}^{m}\left\{\sum_{j=1}^{n_q-1} b_{qj}\frac{1}{(z-z_q)^j} + b_{q0}\ln(z-z_q)\right\} \end{aligned}\right\} \tag{3}$$

Here b_{qi} are certain constants (of which $b_{q,\,n_q-1}$ and b_{q0} are real), and we have $\alpha = i$ or $\alpha = 1$ according as $\sin \mathrm{Re}\,\chi_0(\infty)$ is different from or equal to zero, $\chi_0(z)$ being a function regular in S_0, the real part of which is equal to $\omega(s) - n_0\theta_0$ on L_0 (θ_0 being the polar angle). For the sake of definiteness we shall assume that $\alpha = i$.

Substituting in (1) the expression for $u(x, y)$ from (2) we obtain for the determination of the density $\nu(s)$ an integral equation of Fredholm, which is equivalent to the problem itself ($^{1-5}$) ($r = \sqrt{(\xi-\xi_0)^2 + (\eta-\eta_0)^2}$, and the points $M(\xi, \eta)$ and $M_0(\xi_0, \eta_0)$ correspond to the arc lengths s and s_0):

$$\nu(s_0) + \int_L \nu(s) K(s, s_0)\,ds = F(s_0) \tag{4}$$

$$\left.\begin{aligned} K(s, s_0) &= \frac{1}{\pi}\left[(a(s)a(s_0) + b(s)b(s_0))\frac{\partial \ln r}{\partial n} + \right. \\ &+ (a(s)b(s_0) - a(s_0)b(s))\frac{\partial \ln r}{\partial s} - \mathrm{Re}\left\{(a(s_0) + ib(s_0))\frac{\partial \delta(s, s_0)}{\partial t_0} - \right. \\ &\left.\left. - c(s_0)\left[i(a(s) - ib(s))\frac{dt}{ds}\ln\varepsilon(s)(t - t_0) - \delta(s, s_0)\right]\right\}\right] \\ \delta(s, s_0) &= \begin{cases} -\frac{1}{2}t^{n_0-1}(a(s) - ib(s))\frac{dt}{ds}, & s \text{ on } L_0,\ s_0 \text{ on } L \\ i(b(s)\cos(n, \xi) - a(s)\cos(n, \eta))\ln(t_0 - z_q) & s \text{ on } L_q \\ \qquad (q = 1, \ldots, m), & s_0 \text{ on } L \end{cases} \\ F(s_0) &= f(s_0) - \mathrm{Re}\{(a(s_0) + ib(s_0))g'(t_0) + c(s_0)g(t_0)\} \end{aligned}\right\} \tag{5}$$

§ 2. Let us now show that the determination of the fundamental functions of the adjoined equation corresponding to (4) may also be reduced to a certain problem of Poincaré. In fact, this equation, as may easily be seen, can be written in the form

$$\begin{aligned} &\mathrm{Re}\left[(a(s_0) - ib(s_0))t'(s_0)\left\{-\lim_{z\to t_0}\left[\frac{1}{\pi i}\int_L \mu(s)(a(s) + ib(s))\frac{ds}{t-z} + \right.\right.\right. \\ &\left. + \frac{1}{\pi i}\int_L \mu(s)c(s)\ln\varepsilon(s_0)(z-t)\,ds\right]\Big\} - \\ &\left.\left. - \frac{1}{\pi}\int_L \mu(s)(a(s) + ib(s))\frac{\partial\delta(s_0, s)}{\partial t}\,ds - \frac{1}{\pi}\int_L \mu(s)c(s)\delta(s_0, s)\,ds\right] \end{aligned} \tag{6}$$

where the point z tends to t_0 from the outside of the domain S. Hence, denoting by $\chi_0^*(z)$ a function regular in S_0, the real part of which is equal to $\omega(s) - \theta^* - (n_0 - 1)\theta_0$ on L_0 (θ^* being the angle formed by the normal to L with the x axis), we shall have on L_0

$$\begin{aligned} &\mathrm{Re}\left[ie^{-i\chi_0^*(t_0)}\frac{1}{t_0^{n_0-1}}\left\{-\lim_{z\to t_0}\left[\frac{1}{\pi i}\int_L \mu(s)(a(s) + ib(s))\frac{ds}{t-z} + \right.\right.\right. \\ &\left.\left. + \frac{1}{\pi i}\int_L \mu(s)c(s)\ln\frac{z-t}{z}\,ds\right] + t_0^{n_0-1}\frac{1}{2\pi}\int_L \mu(s)c(s)\,ds\right\}\Big] = 0 \end{aligned} \tag{7}$$

From the last equality, assuming that $\sin \operatorname{Re} \chi_0^*(\infty)$ is different from zero*, we find

$$\int_L \mu(s)\, c(s)\, ds = 0 \tag{8}$$

and, putting

$$\varphi(z) = \frac{1}{2\pi i} \int_L \mu(s)\,(a(s) + ib(s)) \frac{ds}{t-z} + \frac{1}{2\pi i} \int_L \mu(s)\, c(s) \ln(t-z)\, ds \tag{9}$$

we shall have: $\varphi(z) = 0$ in S_0. Similarly we find that $\varphi(z) = d_q$ in S_q $(q = 1, \ldots, p)$ where $d_q = \operatorname{Re} \varphi(z_q)$ $(q = 1, \ldots, m)$ and

$$\varphi(z) = \sum_{k=0}^{n_q - 2} a_{kq} (z - z_q)^k + ie^{i\chi_q^*(z)} (z - z_q)^{n_q - 1} (\delta_q(z) - iC_q)$$

in S_q $(q = p+1, \ldots, m)$, where $\chi_q^*(z)$ is a function regular in S_q, the real part of which is equal to $\omega(s) - \theta^* - (n_q - 1)\,\theta_q$ on L_q (θ_q being the angle formed by the vector $t - z_q$ with the direction of the x axis), $a_{kq} = a_{kq}^{(1)} + ia_{kq}^{(2)}$ ($a_{kq}^{(1)}$, $a_{kq}^{(2)}$ and C_q being arbitrary real constants, $a_{0q}^{(1)} = dq$) and

$$\delta_q(z) = \sum_{k=0}^{n_q - 2} \{a_{kq}^{(1)} \delta_{kq}^{(1)}(z) + a_{kq}^{(2)} \delta_{kq}^{(2)}(z)\}$$

besides $\delta_{kq}^{(1)}(z)$ and $\delta_{kq}^{(2)}(z)$ are also functions regular in S_q; their imaginary parts vanish at z_q and their real parts are respectively equal on L_q to

$$\operatorname{Re} ie^{-i\chi_q^*(t)} (t - z_q)^{k - n_q + 1}, \qquad -\operatorname{Re} e^{-i\chi_q^*(t)} (t - z_q)^{k - n_q + 1}$$

Taking all this into account, we shall have on L

$$\frac{d}{dt}\left\{\mu(s)\,(a(s) + ib(s))\,\frac{ds}{dt}\right\} - \mu(s)\, c(s) \frac{ds}{dt} + \varphi'(t) = \psi(t)$$

where $\psi(z)$ is regular in S. Hence, introducing the function $\psi^*(z) = u^* + iv^*$, the derivative of which is equal to $\psi(z)$, and denoting by

$$a_1 = b \cos(n, \xi) - a \cos(n, \eta), \quad b_1 = b \cos(n, \eta) + a \cos(n, \xi)$$

we obtain on L (assuming $b \neq 0$)

$$\frac{dv^*}{dn} - \frac{a_1}{b_1} \frac{dv^*}{ds} + \left(\frac{c(s)}{b_1} - \frac{d}{ds} \frac{a_1}{b_1}\right) v^* = \frac{c(s)\,(g_2 - A_q)}{b_1} - \frac{d}{ds}\left\{g_1 + \frac{a_1}{b_1}(g_2 - A_q)\right\} \tag{10}$$

where v^* is one-valued in S, $\varphi(z) = g_1 + ig_2$, and the A_q are certain constants $(A_0 = 0)$.

Thus, the determination of $\mu(s)$ is reduced to the solution of Poincaré's problem.

If the function $b_1(s)$ vanishes at the points with the affices α_k $(k = 1, \ldots, n)$, and its derivative at these points is different from zero, then by putting

$$\psi^*(z) + \prod_{k=1}^{n} (z - \alpha_k)\, \Omega(z)$$

we obtain on L an equality of the form $(\Omega = \Omega_1 + i\Omega_2)$

$$a^*(s) \frac{\partial \Omega_1}{\partial S} + b^*(s) \frac{\partial \Omega_2}{\partial n} + c^*(s)\, \Omega_1 + d^*(s)\, \Omega_2 = Q(s)$$

where $a^{*2} + b^{*2} \neq 0$.

* If $\sin \operatorname{Re} \chi_0^*(\infty) = 0$, then putting in (3) $\alpha = 1$ we again arrive at (8). In the right-hand side of the equality (2) we must, generally speaking, introduce a new real constant b_{00}. For simplicity we assume that $n_0 > 1$.

Remark. In the mathematical theory of tides Poincaré was led to the boundary condition

$$\frac{du}{dn}+p(s)\frac{du}{ds}+q(s)u=f(s) \tag{11}$$

In this case, proceeding on similar lines, we obtain instead of (10) the equality

$$\frac{dv^*}{dn}-p(s)\frac{dv^*}{ds}+\left(q(s)-\frac{dp(s)}{ds}\right)v^*=A_q\left(\frac{dp}{ds}-q(s)\right) \tag{12}$$

It may easily be proved that under the condition (11) (for a multiconnected domain) the problem considered is, generally speaking, insolvable.

If the domain S is simply connected ($L_0=L$ and $A_q=0$), then homogeneous problems corresponding to the conditions (11) and (12) ($f(s)=0$), will have simultaneously either only trivial solutions or an equal number of linearly independent non-trivial solutions.

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REFERENCES

[1] H. Poincaré, Leçons de mécanique céleste, III, chap. X, Paris, 1910. [2] G. Bertrand, Ann. Sc. de l'Éc. Norm. Sup., 3 sér., **40** (1923). [3] W. Pogorzelski, Math. Z., **44** (1939). [4] B. Khvedelidze, C. R. Acad. Sci. URSS, XXX, No. 3 (1941). [5] Б. В. Хведелидзе, Сообщ. АН Груз. ССР, II, No. 7, 10 (1941).

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PHYSICS

ON RESONANCE PHENOMENA ASSOCIATED WITH THE MOVEMENT OF A RELATIVISTIC PARTICLE IN THE CYCLOTRON

By A. ANDRONOV and G. GORELIK

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1. As is well known, the acceleration of a charged particle in the cyclotron can be looked upon as a resonance phenomenon. This resonance is n o n - l i n e a r even in the case of a particle of constant mass. In fact, the variable electric field is practically distinct from zero only in the narrow slit between the duants, so that the force acting upon the particle is not merely a periodical function of time, but also a non-linear function of the particle's coordinates. But if the particle is one of constant mass, its own vibrations (motion in the absence of an electric field) can be described as circular vibrations of a l i n e a r oscillator whose proper frequency is the angular velocity of rotation of the particle in the magnetic field, independent of the energy of the particle. In the case of a r e l a t i v i s t i c particle a second non-linearity, the dependence of the mass on the velocity, is superimposed upon the above-mentioned one. According to the oscillation theory, here, too, resonance phenomena, though of essentially different type, should take place. Some characteristic features of those phenomena the authors were able to elucidate by analysing the periodical motions that establish themselves—at least theoretically—owing to loss of energy by radiation and collisions. Particularly, we should like to call attention to the fact that in the case of a relativistic particle the resonance curves (energy of periodical motion plotted against the magnetic field H) resemble curves of ferro-resonance in exhibiting for sufficiently large amplitudes of the disturbing force a «slant» and a hysteresis loop.

This is only natural, indeed, seeing that a relativistic particle in the cyclotron presents an oscillator whose frequency of proper (circular) oscillations depends upon its energy, just as the frequency of proper oscillations of a circuit with an iron core coil. The existence of such a resonance curve implies the possibility to «pull» the particle into the region of large energies by gradually increasing the magnetic field. We may as well point out the possibility of frequency demultiplication. This is connected with the f i r s t of the above-mentioned non-linearities.

Our interest in the problem of acceleration of relativistic particles in a cyclotron was aroused by two papers of Veksler ([1],[2]) who among other things suggested the increasing of H as a method for the acceleration of the particles. But our approach to the problem (investigation of the stationary state) is essentially different from his.

2. Supposing the motion of the particle in the cyclotron to proceed along a spiral, denote by t_n the instant when the particle (charge e) for the n-th time crosses the accelerating slit (assumed to be infinitely narrow), leaving it with energy E_{n+1}.

Let $V \sin \omega t$ be the voltage applied to the slit; $f(E)$, the energy lost by the particle in running half-way around when it leaves the slit with energy E.

The balance of energy is described by the equation

$$E_{n+1} - [E_n - f(E_n)] = eV \sin(\omega t_n + n\pi) \qquad (1)$$

On the other hand,

$$t_{n+1} - t_n = \pi \frac{\overline{E}_{n+1}}{eHc} \qquad (2)$$

where c is the velocity of light; $\overline{E}_{n+1}$ is the average value of energy possessed by the particle at some time between the n-th and the $(n+1)$-th passsage through the slit. We take that

$$\overline{E}_{n+1} = E_{n+1} - \frac{1}{2} f(E_{n+1}) \qquad (3)$$

We shall look for such solutions of the system of non-linear equations (1)—

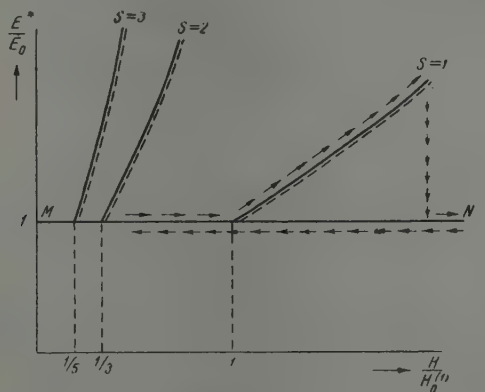

Fig. 1

(3) for which E_n has the same value E^*, whatever the value of n. From (1) we have

$$f(E^*) = eV \sin(\omega t_n + n\pi)$$

whence

$$t_n = t_0 + n(2s-1)\frac{\pi}{\omega} \ (s \text{ is an integer}), \quad f(E^*) = eV \sin \omega t_0 \qquad (4)$$

With $s=1$ we have a periodical motion having the same period with the electric field. With $s>1$ we have frequency demultiplication. Furthermore

$$E^* - \frac{1}{2} f(E^*) = \frac{eHc}{\omega}(2s-1) \qquad (5)$$

With s given, the equation (5) uniquely determines the value E^* for every value of $H > H_0^{(s)}$, where $\frac{eH_0^{(s)}c}{\omega}(2s-1) = E_0$; E_0 is the rest energy.

For $f(E^*) < eV$ equation (4) gives two real values which merge into a single one for $f(E^*) < eV$; for one of them $\cos \omega t_0 > 0$, for the other $\cos \omega t_0 < 0$. With $f(E^*) > eV$, t_0 assumes imaginary values, which means that there is no periodical solutions of the type we are looking after.

For investigating the stability of the periodical solutions put $t_n = t_0 + n\frac{\pi}{\omega} + \tau_n$. Linearising (1), (2), (3) with respect to τ_n, we get

$$\tau_{n+1} - 2\tau_n + \tau_{n-1} + \frac{k}{2}(\tau_{n+1} - \tau_{n-1}) = \frac{\pi eV}{Hc} \cos \omega t_0 \, \tau_n \qquad (6)$$

where $h = \frac{\pi}{eHc} f'(E^*)$. In view of the smallness of f we may as an approximation put $E^* = \frac{eHc}{\omega}(2s-1)$.

Equation (6) gives $\tau_n = ae^{n\lambda}$, where

$$e^{\lambda} + e^{-\lambda} + \frac{h}{2}(e^{\lambda} - e^{-\lambda}) = 2 + \frac{\pi\omega V}{Hc}\cos\omega t_0$$

Physical conditions require that $eV \ll E^*$, *i. e.* $\pi\omega V \ll Hc$. If $\cos\omega t_0 < 0$, the real part of λ is negative (the periodical solution is stable), and if

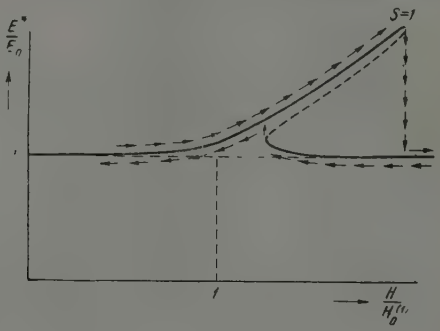

Fig. 2

$\cos\omega t_0 > 0$, there is positive root λ (the periodical solution is unstable). So one of the periodical solutions is stable, the other unstable.

3. The obtained dependence of E^* upon H is represented in Fig. 1 for $s = 1, 2, 3$. The right line MN represents the state of rest (energy $E = E_0$). The highest attainable energies are defined by the condition $f(E^*) = eV$; generally speaking, $f(E^*)$ depends also on H.

A more general theory, evolved for a slit of finite width, gives a curve which for $s = 1$ is represented in Fig. 2.

Its resemblance with the curve of ferro-resonance is more obvious (the unstable branch is indicated by dots), and its limiting case is the curve for $s = 1$ in Fig. 1. The arrows indicate the run of the process when H increases or diminishes monotonously.

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REFERENCES

[1] V. I. Veksler, C. R. Acad. Sci. URSS, XLIII, 346 (1944). [2] V. I. Veksler, C. R. Acad. Sci. URSS, XLIV, 393 (1944).

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GEOPHYSICS

THEORETICAL AND ACTUAL TOTAL HEAT OF SOLAR RADIATION FOR CLOUDLESS SKY

By N. N. KALITIN

(Communicated by A. A. Grigoriev, Member of the Academy, 20. VI. 1945)

There are two kinds of theoretical total heat of solar radiation reaching the Earth surface.

1) The first is evaluated from astronomical data, *viz.* the declination of the Sun and the latitude of the place of observation making no allowance for the influence of the atmosphere. These values can easily be computed and in this country such tables of total heats have recently been compiled by S. I. Savinov. In these tables diurnal total heats of solar radiation expressed in calories per square centimetre of horizontal surface are given for each day of the year and for latitudes ranging from 0 to 90°.

2) To the second kind belong the diurnal heat supplies by solar radiation computed theoretically for an ideal atmosphere supposed to be perfectly dry and pure. These are the highest possible totals which can only be received by the Earth surface under the most favourable atmospheric conditions. These totals have recently been computed by N. G. Evfimov for each day of the year for different latitudes.

It seems interesting to compare the available theoretical data of total heat supplied by the Sun with those measured actually at a particular point of the Earth surface. Thereby we shall form an idea as to the radiation properties of the real atmosphere under different climatic conditions.

In order to derive reliable average data from observations we must have at our disposal records of solar radiation intensity taken continuously through a large number of years, because in selecting the material for treatment we can only choose data obtained on perfectly cloudless days. Cloudless days, however, are scarce in certain months, even under most favourable climatic conditions.

The material used in the present paper has been selected from observations made at Pavlovsk for a period of 28 years (1913—1940). Long as is this period (and no longer records have been made at any point of the globe), it was found to include only 718 cloudless days suitable for treatment (Table 1).

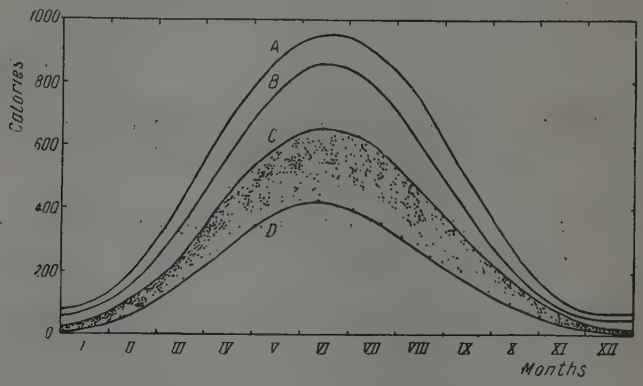
Table 1

Number of Cloudless Days at Pavlovsk for 28 Years

Months	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Total for the year
Number of days . . .	34	64	83	76	90	72	77	65	34	37	23	43	718

The total number for different months ranges from 90 days in May down to 23 in November. It must be emphasized that selected for treatment were not only those days on which the sky has shown no signs of clouds throughout the day, but also those during which the record of the actinograph was never disturbed by clouds, and which therefore can be looked upon as cloudless as regards heat supply by solar radiation.

If we were to confine ourselves to perfectly cloudless days, their number under the climatic conditions prevailing at Pavlovsk would be found to have been con-



siderably smaller, and in spite of the 28 years of continuous record there would be no material for reliable conclusions.

It is of interest not merely to know the average monthly total of solar radiation for each month and to compare it with the respective theoretical values, but also to determine the deviations from the average. The reason for these deviations will be found to lie mainly in the variation of the content of water vapour and dust and to have an unmistakable tendency towards decrease rather than increase of the radiation inflow.

In the adjoined figure all of the 718 observed values of diurnal total heats of solar radiation, expressed in calories per square centimetre of horizontal surface, are plotted against the different months of the year. Every point gives the daily total. The variations in the total heat of solar radiation due to the varying transparency of the atmosphere are reflected in the scattering of the points along the vertical. The scattering is greatest (in absolute value) for the summer months, which fact indicates that it is at this time that the total heat supply by solar radiation is the most strongly affected by variations in the transparency of the atmosphere.

Curve *C* is drawn through the maxima, and curve *D* through the minima of the observed total heats. Thus, all the observed total heats are comprised within the area between these curves.

Computation shows that the scattering of the points is subject to pronounced yearly fluctuations, being least in the spring (March, 39 per cent) and greatest in the winter (December, 100 per cent).

The two other curves in the drawing bring out the yearly variations of the total heat in the absence of atmosphere, according to Savinov (curve *A*), and for an ideal atmosphere, according to Evfimov (curve *B*). Comparing the observational data with these curves one can get an insight into the influence of the atmosphere on the supply of heat by solar radiation.

An example of such a comparison is afforded by Table 2. Under *a* and *b* are listed the monthly averages of diurnal total heats taken from the theoretical curves of the figure, and under *c*, the averages over all points for each month.

Table 2

Mean Total Heats of Solar Radiation for Different Months

Months	No atmosphere	Ideal atmosphere	Observation	$\frac{c}{a} \cdot 100$	$\frac{c}{b} \cdot 100$
	a	b	c	d	e
January	100	60	35	35	58
February	210	160	100	48	62
March	420	340	230	55	68
April	670	560	390	57	70
May	880	760	510	58	68
June	980	840	560	57	67
July	920	790	510	56	65
August	740	620	400	54	64
September	500	410	260	52	63
October	280	220	130	46	59
November	130	85	50	38	59
December	80	45	20	25	44

It may be seen from Table 2 that the influence of the atmosphere in decreasing the inflow of radiated heat is strongest in the winter season (the minimum of the ratio between the observed and the theoretical total heat falls on December—25 and 44 per cent). This is not merely because at this time of the year the Earth surface at Pavlovsk is least directly under the Sun's rays, whereby the influence of the atmosphere is greatly enhanced in general, but also because the air is more turbid at low temperatures.

As was pointed out above, comparison between observed and theoretical means of total heat supplied daily by solar radiation is not possible unless we possess data gathered by continuous observations during a period of many years. Experience gained in treating the observations made at Pavlovsk shows that even as long a period as 28 years does not provide a sufficient basis for reliable conclusions relative to some of the months (November).

But this is only true of localities with a climate similar to that prevailing at Pavlovsk (seasons with dense cloudiness). For places with a more sunny climate the observations may cover a considerably shorter period. In this country we have several actinometric stations in the south, where cloudless days are of frequent occurrence. Valuable data can be brought to bear on the subject if these stations are involved in studies similar to those reported in the present paper.

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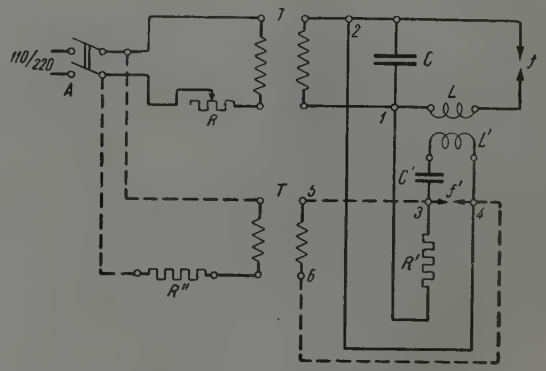
TECHNICAL PHYSICS

A SPARK GENERATOR WITH FORCED IGNITION FOR SPECTROSCOPIC ANALYSIS

By F. A. KOROLEV

(Communicated by S. I. Vavilov, Member of the Academy, 5. IX. 1945)

In exciting spectra with spark generators, one is anxious to ensure maximum reproducibility of the electrical conditions and to apply some or other method for controlling the spark discharge. This is the case in the spark generator of Feissner



and, partly, in the spark generator with two successive sparks. Both the first and the second types of generator, however, have a number of essential shortcomings.

In this paper is proposed a new type of spark generator, with forced ignition, for spectroscopic analysis. It can be designed in two ways.

In the first design (see figure), the main circuit of the spark generator consists of the following parts: a sparker f with the test samples, connected through a high-frequency coil L (of the order of 10^{-5} cycles) to a high-voltage condenser, C, which is connected in turn to a high-voltage winding (1000—10 000 V) of a step-up transformer T. The latter is fed by 110/220 V line through a suitable rheostat R. The sparker f is so adjusted that there is no breakdown with the given voltage applied to the electrodes. Forced ignition of the discharge is effected by an auxiliary low-power circuit $L'\,C'\,f'$ fed through a high-voltage (glass-inclosed) resistance of the order of 50 000 Ω. The capacity of condenser C' is approximately ten times less than that of condenser C. When the auxiliary sparker f' is of suitable magnetude, it gives a steady low-power spark discharge. The discharge impulses are

put into the circuit LCf through the coupled coils L' and L, and bring about the forced ignition of f.

In the second method (which is convenient to employ when low voltages are impressed on the secondary winding of the transformer T), forced ignition is brought about with the aid of an ordinary activator (oscillator) of an alternating current arc. The figure shows, how to change from the first method to the second. It is done by disconnecting the resistance R' from terminals *1* and *3* and also removing the connexion with terminals *2* and *4*. Instead, terminals *3* and *4*, are connected, respectively, to terminals *5* and *6* (this connexion is indicated in the figure by dotted lines) of the high-voltage winding of a low-power step-up transformer T', which usually serves as an activator of an alternating current arc ($\sim$30 VA). The primary winding of the transformer is connected through a resistance R'' to a 110/220 V line. Not until the activator is switched on, does a discharge appear in the sparker f, whereby a forced ignition is effected.

Both in the first and second method an exceptionally steady spark regime may be maintained in f by regulating the sparker f'.

A big advantage of the proposed circuit for a spark generator with forced ignition is the fact that the electrical regime is controlled by a low-power auxiliary circuit which can be controlled very carefully. Since the sparker f' does not practically wear off in the process of operation, there is no need for further regulation of the regime.

A second highly important advantage of the circuit is its ability to operate at any desired voltage (1000—12 000 V), a feature that does not figure among the characteristics of other types of generators.

Both of the forced ignition generator circuits proposed here were put to the test; the tests were run with 6000 V applied to the secondary winding of the transformer T (of the NOM-6 type), $C=0.02$ μF; and with 1000 V, $C=1$ μF.

In both cases the relative intensity of the spectral lines at very high spectrum brightness showed a very high stability in the course of time. The tests were made both photographically and visually with the aid of a stilometer.

The results of this investigation revealed the possibility of effectively utilizing the entire voltage range from 1000 V outward, which very considerably broadens the prospects of spark excitation of spectra.

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CHEMISTRY

ON AREA MEASUREMENTS IN THERMOGRAMS FOR QUANTITATIVE ESTIMATIONS AND THE DETERMINATION OF HEATS OF REACTION

By L. G. BERG

(Communicated by I. I. Chernjaiev, Member of the Academy, 18. I. 1945)

Quantitative thermographic phase analysis and the determination of heats of reaction by the method of heating curves involves the measurement of the areas on the thermograms formed by the deviation of the differential record from

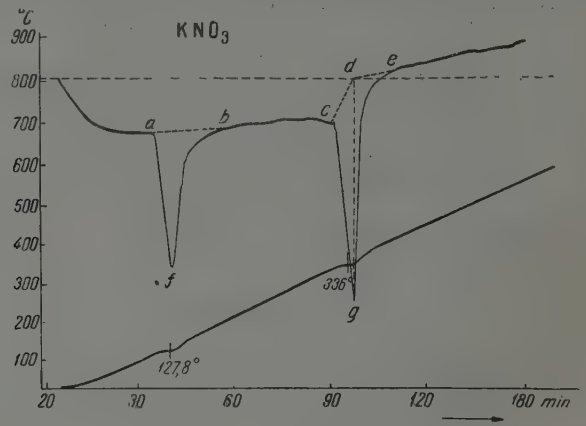

Fig. 1

the direction taken in the absence of the effect ([1-3]). If the differential curve after the effect is the direct continuation of the record taken before, the upper boundary of the area is obtained very simply: a straight line is drawn connecting the points where the deviation of the differential record begins and ends (Fig. 1, *ab*).

Sometimes, however, in the process of reaction an appreciable change may take place in the conditions of heat transfer in the sample. Loss in weight of the sample during dissociation or dehydration should lead to a decrease in the specific heat and, consequently, to a corresponding elevation in the temperature of the weighed sample. On the differential curve this leads to an upward deviation of the record*. Further on the record may proceed parallel to its initial direction, but on a different level, *i. e.* at a different distance from the zero line.

* In our set-ups upward deviation of the differential curve denotes heating of the sample, and downward deviation, cooling and endothermal effects.

The differential curve is affected still more by the change in thermal conductivity during formation of a new phase, or transition into another state of aggregation.

For example, after the decomposition of some platinum complex into metallic platinum, the level of the differential record rises abruptly, owing to the appreciably greater thermal conductivity of the metal. The same thing is sometimes ob-

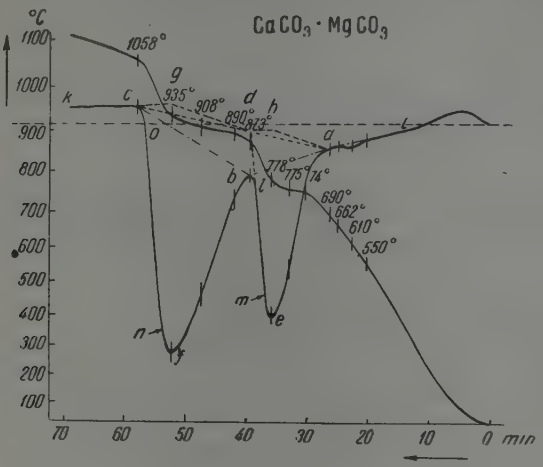

Fig. 2

served during the fusing of salts: the appearance of a liquid phase and convection of the molten mass gives rise to an increase in heat transfer (Fig. 1, *e*).

Here it is proposed to examine a method of defining the boundaries of endothermal effect area, whereby the latter if necessary can be measured with a planimeter.

If the process is accompanied by a change in heat transfer, *i. e.* the differential record after the effect is not a direct continuation of the previous curve, this change should make itself manifest only in the first stage of the effect.

Then a perpendicular is erected to the time axis at the maximum of the deviation of the differential curve, which corresponds to the completion of the process, and the linear direction of the differential record that established itself after the reaction, is produced to intersect the perpendicular (Fig. 1, *d*—*e*); the point of intersection *d* is then connected by a straight line with the point where the differential curve begins to deviate (point *c*, Fig. 1), which in turn is determined by the point where the record begins to deviate from the established linear direction.

With such a method of delimitating the areas, the increase in quantity of the new phase with a different thermal conductivity and the decrease in quantity of the previous phase are indicated on the thermogram by an inclined line in the course of the process itself (Fig. 1), since the return of the curve to its previous direction corresponds only to equalization of the temperatures of the sample and the standard after completion of the reaction.

Another difficulty in measuring the areas consists in separating the areas of two overlapping effects, when the differential curve begins showing a new deviation corresponding to the second effect, before it has time to return its previous position (before reaction) after the first deviation. In this case if the points where the deviation of the differential record begins and ends (Fig. 2, *a* and *c*) are joined to the point where the second deviation begins (Fig. 2, *b*), an appreciable area (*abcd*) is found to be unmeasured. The greater the overlapping, *i. e.* the larger the total area (*abcd*), the less accurate is the ratio between the areas obtained for the two effects (*aeb* and *bfc*).

It is often possible to appreciably decrease and sometimes completely eliminate overlapping of the areas of the two effects either by heating slower, or decreasing the weight of the sample. It is fully obvious that if superposition of the effects persists, the entire area formed by the deviation of the differential record must be measured. The question arises, How to separate the areas obtained.

Let us suppose that there was no second effect; then we, first, bound the entire area from above by a straight line (*ac*, Fig. 2), as described in the foregoing, and draw a second straight line (*ld*, Fig. 2), which is the continuation of the linear return portion of the differential record after the first effect, until it intersects the first line (*ac*).

Thus, the area of the first effect is obtained. All the rest of the area (*lbfcd*) will correspond to the second effect. Experiments with dolomites, which always give slightly overlapping effects, showed that computation of the areas by this method is correct.

In cases of marked overlapping, the curve of temperature equalization may not have a linear portion (*ml*). Then a straight line is drawn from the point where the second deviation starts (*b*, Fig. 2) parallel to the linear portion of the temperature equalization curve after the second effect. This can be done because the linear portions of the equalization of the temperatures of the sample and the standard are approximately parallel for both effects.

It is obvious that the greater the overlapping of the effects, the less accurate will be the results obtained, though not to such an extent as to preclude the use of the method. For greater accuracy it is necessary, of course, to proportion the rate of heating to the quantity of the weighed sample. The measured areas will be even still more accurate if the changes in thermal conductivity in the course of the processes are simultaneously taken into account, as pointed out in the foregoing (Fig. 2, areas *aeldh* and *bfcgd*), although in this case we make the assumption that the change in thermal conductivity is distributed proportionally to both effects, which may actually not be exactly so.

In Table 1 are given data on the composition of Samara Luka dolomites as determined by measurements of the areas and by chemical analysis. The mean accuracy of thermographic analysis at the present stage fluctuates within the limits of 2—3 per cent of the weighed sample, which is fully acceptable for rapid quantitative phase analysis. To determine reaction heats this method is the simplest and quickest.

Moreover, the accuracy of a determination may be increased if the mean of several determinations is taken, and various substances are utilized as comparison standards.

Table 1

Chemical analysis, mol. %		Thermographic analysis, mol. %			
Calcite	Dolomite and magnesite recomputed in terms of dolomite	Calcite (900°)	Dolomite (730°)	Magnesite (600°)	Discrepancy (±) per cent of calcite (I—III)
I	II	III	IV	V	VI
27.4	72.6	27.2	70.2	2.6	−0.2
27.4	72.6	26.9	73.1	—	−0.5
26.0	74.0	26.7	72.7	0.6	+0.7
69.3	30.7	69.4	30.6	—	+0.1
64.8	35.2	66.9	33.1	—	+2.1
64.8	35.2	62.5	37.5	—	−2.3

If the sample contains appreciable quantities of magnesite, thermographic analysis usually gives a larger calcium content, the excess of $CaCO_3$ being close to the content of free magnesite. The content of dolomite should naturally be less,

since in the chemical analysis magnesite was estimated with a corresponding quantity of $CaCO_3$ in the form of dolomite.

Table 2

I	II	I	IV	V	VI (I—III)
36.5	63.5	43.2	49.3	7.5	+6.7
41.0	59.0	45.2	49.5	5.3	+4.2
31.2	68.8	34.6	62.9	2.5	+3.4

If the last three tests are recomputed without magnesite, the difference between the data of the thermographic analysis and chemical analysis does not exceed 2 per cent.

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REFERENCES

[1] Л. Г. Берг и Ф. Ф. Лаптев, Тр. МГРИ, **20**, 141 (1941). [2] L. G. Berg and I. N. Lepeshkov, C. R. Acad. Sci. URSS, XXXI, No. 6 (1941). [3] Л. Г. Берг и В. Л. Аниссов, ЖОХ, **12**, 31 (1942).

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CHEMISTRY

THE EFFECT OF PROMOTERS IN THE CATALYTIC DEHYDROGENATION OF ALCOHOLS TO ESTERS AS RELATED TO THEIR PLACE IN THE PERIODICAL TABLE

By S. L. LELCHUK

(Communicated by A. N. Nesmeyanov, Member of the Academy, 10. I. 1945)

The author has been engaged in a systematic study of various elements and their oxides as promoters in the catalytic dehydrogenation of alcohols to esters. The reactions that take place over the catalyst can be represented schematically by the following equations

$$1.\ R-CH_2OH \rightarrow R-CHO + H_2$$
$$2.\ R-CHO + H_2O \rightarrow R-COOH + H_2$$
$$3.\ R-CH_2OH + R-COOH \rightarrow R-CH_2OOCR + H_2O$$

The first of these reactions, *viz.*, the one that yields an aldehyde, is well known, and dates back, indeed, to the time when the first catalytic heterogeneous reactions were discovered. The second reaction, which gives birth to an acid, became known considerably later, and is studied at present by several different workers and by the author himself. The third reaction, which results in the formation of an ester, has been thoroughly investigated by the author(1). He has found that copper when promoted by certain elements is an active catalyst in this reaction. The promoting effect was studied on precipitated as well as alloy catalysts. The precipitated catalysts consisted, as a rule, of copper containing a finely distributed oxide of the respective metal.

It may be seen from Table 1 that reaction 3 is activated by Ce in the III group and by elements of the even series in the groups IV, VI and VIII. The elements of the odd series and groups fail to activate Cu for reaction 3, though some of them, *e. g.* Zn, Tl, La and Mn, enhance reaction 1 (formation of aldehyde). It may be noted that reaction 1 is the more enhanced, the greater the atomic weight of the promoter (see, for instance, group III in Table 1).

Increase in the atomic weight of the promoter has a favourable effect on reaction 3, too (see groups IV and VI). So the desired effect can be obtained with a smaller amount of the promoter. The best results, for example, are obtained with 4.0 per cent of Ti, or 0.9 per cent of Zr, or 0.1—0.2 per cent of Th. Further investigation has revealed that active in every group are the elements of the even series. Of the elements investigated by the author, Ti, Zr and Th in the IV group, Cr and U in the VI, and Co and Ni in the VIII proved to be active. A feature characteristic of these elements is an even number of electrons(2) at the valence energy level, and in a vast majority of the promoters the number of electrons at the next energy level is likewise even.

Table 1

Dehydrogenation of Ethyl Alcohol over Precipitated Catalysts

Promoter	Per cent by weight in Cu	Conversion of alcohol in per cent		Note
		to ester	to aldehyde	
II group				
Be	1.8	9.9	30.50	
Mg	1.0	4.8	33.9	
Zn	0.5	6.0	37.9	
III group				
Y	0.2	6.4	31.3	
Tl	1.0	6.7	41.7	
La	0.2	10.7	45.4	
Ce	0.2	33.7	37.6	
IV group				
Ti	4.0	28.6	33.1	
Zr	0.9	39.8	32.5	
Th	0.2	34.3	40.0	
V group				
V	0.2	1.11	—	
VI group				
Cr	2.0	23.6	47.9	
U[3]	0.1	55.7	44.8	
VII group				
Mn	4	6.1	40.9	
VIII group				
Co**	0.5	16.6	33.2	
Ni***	0.5	14.0	25.0	
Without promoter	—	12.33	36.0*	

* The yield of aldehyde is recalculated according to reactions 2 and 3, and gives the primary decomposition of alcohol. The analytically determined amount of aldehyde is less than that referred to in the table.

** Intense formation of methane.

*** Most intense formation of methane.

In the case of Co and Ni the formation of methane is due to disintegration of acetaldehyde under these conditions.

The conclusions derived from the results obtained were extended to alloy–skeleton—catalysts. In this case, unlike that of precipitated catalysts, we had to deal with metals instead of with a metal and an oxide. The most effective promoters among the precipitated catalysts were tested on alloys. The results of these tests are given in Table 2.

It was found, in fact, that every one of the elements that activate reaction 3 as oxides is also active when used as a metal. The results of this investigation are brought out by the adjoined figure where the elements active in reaction 3 are enclosed in heavy frames, and those failing to promote it stand in light frames. The regularity is most conspicuous in the fourth series, where Ti (group IV) is active, V (group V) is inactive, Cr (group VI) is active, Mn (group VII) is inactive, Co and Ni (group VIII) are active, but give rise to several side reactions. Thus, we can anticipate activity to reaction 3 in, say, Hf, Mo, W and Pd.

In my opinion, this behaviour of the elements of the even series is connected with their particular properties. It is well here to recall the following statement of Mendeleyev[2]; «The elements of the even series form the most active bases, and the basic property in a given group increases with the atomic number». Reaction 3 is a reaction of condensation activated by bases. We thus find a natural reason

Table 2

Dehydrogenation of Ethyl Alcohol over Alloy Catalysts

Promoter	Per cent by weight in Cu	Conversion of alcohol in per cent		Note
		to ester	to aldehyde	
Ti	5.0	36.5	42.2	See note to Table 1
Cr	3.0	40.3	40.6	
Zr	0.5	33.9	40.9	
Without promoter	—	31.2	29.1	

to account for the two observations reported here: 1) the promoting effect of the elements of the even series and even groups, and 2) the decrease in the effective amount of promoter with increasing atomic weight.

Periods	Series	Groups of elements										
		I	II	III	IV	V	VI	VII	VIII			0
1	1	H 1 1,008										He 2 4,003
2	2	Li 3 6,940	Be 4 9,02	5 B 10,82	6 C 12,010	7 N 14,008	8 O 16,000	9 F 19,00				Ne 10 20,183
3	3	Na 11 22,997	Mg 12 24,32	13 Al 26,97	14 Si 28,06	15 P 30,98	16 S 32,06	17 Cl 35,457				Ar 18 39,944
4	4	K 19 39,096	Ca 20 40,08	Sc 21 45,10	Ti 22 47,90	V 23 50,95	Cr 24 52,01	Mn 25 54,93	Fe 26 55,85	Co 27 58,94	Ni 28 58,69	
	5	29 Cu 63,57	30 Zn 65,38	31 Ga 69,72	32 Ge 72,60	33 As 74,91	34 Se 78,96	35 Br 79,916				Kr 36 83,7
5	6	Rb 37 85,48	Sr 38 87,63	Y 39 88,92	Zr 40 91,22	Nb 41 92,91	Mo 42 95,95	Ma 43 —	Ru 44 101,7	Rh 45 102,91	Pd 46 106,7	
	7	47 Ag 107,88	48 Cd 112,41	49 In 114,78	50 Sn 118,70	51 Sb 121,76	52 Te 127,61	53 J 126,97				Xe 54 131,3
6	8	Cs 55 132,91	Ba 56 137,36	La 57 138,92	Hf 72 178,6	Ta 73 180,88	W 74 183,92	Re 75 186,31	Os 76 190,2	Ir 77 193,1	Pt 78 195,23	
	9	79 Au 197,2	80 Hg 200,61	81 Tl 204,39	82 Pb 207,21	83 Bi 209,00	84 Po (210)	85 —				Rn 86 222
7	10	— 87	Ra 88 226,05	Ac 89 (227)	Th 90 232,12	Pa 91 231	U 92 238,07					
		Lantanides 58–71										
		Ce 58 140,13	Pr 59 140,92	Nd 60 144,27	61	Sm 62 150,43	Eu 63 152,0	Gd 64 156,9				
		Tb 65 159,2	Dy 66 162,46	Ho 67 163,5	Er 68 167,2	Tu 69 168,4	Yb 70 173,04	Cp 71 174,99				

This conclusion accords well with the mechanism of esterification suggested earlier by the author, which in its final stage proceeds through a reaction of condensation.

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REFERENCES

[1] С. Л. Лельчук, Диссертация ИОХ АН СССР, 1940. [2] Д. И. Менделеев, Периодический закон химических элементов, М., 1934, стр. 53. [3] П. И. Иванников и Е. Гаврилова, ЖПХ, 11, 981 (1938).

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PHYSICAL CHEMISTRY

ON THE RÔLE OF THE PRINCIPLE OF CONSERVATION OF THE VALENCE ANGLE IN THE MULTIPLET THEORY OF CATALYSIS

By A. A. BALANDIN, Corresponding Member of the Academy, and J. T. EIDUS

According to the multiplet theory of catalysis(1), when a molecule interacts with a catalyst it is held on to the surface of the latter by two, three and more active centres (doublet, triplet, multiplet). Herewith certain bonds in the molecule are ruptured, and new bonds between the atoms of the molecule and the active centres of the catalyst are formed. This process of bond rupture or formation may or may not run to completion, and it is this latter case of deformation that is important for catalysis. It involves the formation of an intermediate multiplet complex, which the multiplet theory considers.

This interaction between the particle and the surface (in adsorption or catalysis) implies the existence of a spatial or geometric correspondence between the shape and dimensions of the reacting particle, on the one hand, and the structure and internuclear distances of the lattice of the heterogeneous catalyst, on the other.

This was taken into account by the multiplet theory which establisthed the existence of a definite relation between the catalytic activity of a metal and its atomic radius in the dehydrogenation of six-membered rings and hydrogenation of benzene and its derivatives. This relation, sometimes called «Balandin's square of activity» in the literature, was fully confirmed by an abundant experimental material (2,3).

According to theory, only metals with face-centered cubic and hexagonal lattices—Pt, Pd, Ir, Rh, Cu, Co, Ni, Os, Ru—having atomic radii from 1.385 to 1.244 Å, as well as solid solutions of these metals in one another, may be catalytically active. The limitation imposed upon the atomic radii is connected with the possibility of fitting a six-membered ring of the molecule upon a «sextet» of atoms of the catalyst. Striking confirmation of the multiplet theory was produced recently.

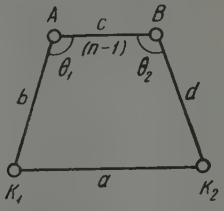
It was found that rhenium is a good catalyst for the dehydrogenation of cyclohexane(4); at the same time rhenium actually falls into the «square of activity», since it has a hexagonal lattice and atomic radii of 1.362 and 1.377 Å (5).

At the present time it is possible to refine the picture at certain points. On the quantum mechanical theory of reaction rates(6), a multiplet complex is an activated complex.

The latter, as is known, is in every way similar to a stable molecule, except for instability in the direction of the reaction coordinate. One of the conditions for the stability of molecules is minimum stress at the valence bonds and valence angles in the generalized sense of Baeyer, measured as their deviation from the normal value. Therefore, in an activated complex there should be minimum stress at definite valence bonds and valence angles in conjunction with instability

along the reaction coordinate. This condition is fulfilled in the sextet model of the multiplet theory.

In unsaturated bond reactions, the principle of minimum stress applies mainly to the valence angles and hence may be called the principle of conservation of valence angles. For chemisorption, hydrogenation and atomic exchange with



deuterium of olefines and also of acetylene compounds on the surface of metals, this principle is corroborated by all of the available experimental data.

The principle of conservation of the valence angle makes it possible to find the optimum atomic radii of catalysts from the multiplet doublet model. The adjoined figure shows such a model of the hydrated part of a doublet complex for the case of the chemisorption of an unsaturated molecule AB with an n-fold covalent bond between atoms A and B ($n=2$ or 3) held down by two atoms of the catalyst K_1 and K_2.

During chemisorption, the n-fold bond $A_{(n)}B$ opens up, passing into an $(n-1)$-fold bond $A_{(n-1)}B$ of length c; the internuclear distances $K_1—K_2$ are equal to a, and the lengths of the new bonds $A—K_1$ and $B—K_2$ formed in the multiplet complex are equal to b and d, respectively. According to the above-mentioned principle, the angles θ_1 and θ_2 in the system ABK_2K_1 should differ little in magnitude from the valence angles of the A and B atoms in the given compound. For given values of c, b, d, θ_1 and θ_2 strictly determined by the structure and configuration of the molecule AB to be hydrogenated, the quantity a is defined geometrically in a unique manner. There is ground for believing that the configuration of the skeleton of the molecule to be hydrogenated in the multiplet complex (transition state) is very much like the configuration of the molecule with simple bond. Therefore, the obtained value a_{opt} will be approximately the optimum internuclear distance in the crystal lattice of the active metal. The optimum atomic radius will be $a_{opt}/2$. It is evident that the greater the distances a in the metal differ from the value a_{opt}, the greater the deformation of the valence angles and the stress in the system, and the more difficult it becomes for the processes of interaction of the particles with the surface of this metal to proceed.

The valence angle was first taken into account by Twigg and Rideal[7] while analysing the doublet diagram for the chemisorption of ethylene on Ni, and then by Herington[8] for the analogous diagram for acetylene. However, they applied the principle of conservation of valence angles to the end product of chemisorption and not to the transition state. This is not correct in general. The numerical error in the case of hydrogenation is not great owing to the similarity in configuration of the transition state and the adsorption product.

The doublet chemisorption of ethylene on a metal was taken by Polanyi as the first stage in his association mechanism of catalytic hydrogenation and atomic exchange with deuterium D_2 of unsaturated compounds. Of the two mechanisms, dissociation and association, suggested for these reactions, the latter may be regarded to fit best the experimental data obtained in a large number of investigations on the catalytic hydrogenation and hydrogen exchange of unsaturated compounds[9,10].

An idea of the doublet model for the chemisorption of ethylene on Ni may be obtained by replacing in the accompanying figure the bond $A_{(n-1)}B$ by a simple C—C bond and the K atoms by Ni atoms and by substituting $\theta_1=\theta_2=109°28'$; $b=d=(C—Ni)=1.82$ Å; $c=1.52$ Å. Then $a_{opt}=2.73$ Å[7]. For acetylene[8] it is found from the doublet model that $a_{opt}=3.20$ Å. In the crystal lattice of Ni, which has

a face-centered cubic structure, two internuclear distances—3.50 and 2.47 Å—are encountered. The first is close to a_{opt} for acetylene, and the second, to a_{opt} for ethylene. Hence C_2H_2 molecules should be chemisorbed easier on the crystal plane containing more Ni—Ni bonds 3.50 Å in length; and C_2H_4 molecules, on the crystal plane with a larger number of bonds 2.47 Å in length.

The conclusion that the active centres forming parts of the different planes of one and the same crystal may have varying adsorption and catalytic activity, owing to the difference in their structure and internuclear distances, is fundamental in the multiplet theory and, as we saw, is refined by taking into consideration the influence of the valence angle. This conclusion is corroborated by a number of experimental data. Thus, it was found that in the hydrogenation of olefines on nickel the (110) and (111) planes have different activity ([7],[11]).

Everything that we have discussed here applies to multiple bonds. For simple bonds the limitations imposed by the principle under consideration are lifted, owing to the univalency of the radicals formed which are bound with the surface. However, the theory of the transition state shows that the internuclear distances play an important rôle here as well, for example, in the hemisorption of a hydrogen molecule.

Thus, it was computed ([12]) that the 3.5 Å internuclear distance, of the two mentioned above, is the optimum for the activated adsorption of hydrogen on nickel.

It is necessary to point out that the multiplet (doublet) model of the adsorption of H_2 by two Ni atoms forms the basis for these computations.

It goes without saying that the stereo-factors are not the only ones influencing the adsorption and catalytic activity of the surface. Highly important are the energy factors([1]) connected with them. Both the former and the latter depend upon the chemical nature of the atoms.

The principle of conservation of the valence angle makes it possible in the case of hydrogenation of ethylene compounds to supplement the list of catalysts that fall into the «square of activity» and are suitable for the hydrogenation of aromatic compounds with metals W, Mo, V, Cr and Fe that crystalize in a volume-centered cubic lattice, whose atomic radii (from 1.367 to 1.239 Å) also lie within the required limits.

These fine distinctions, predicted by the theory, are actually observed. Thus, it was found([3]) that the iron catalyst used in the synthesis of ammonia facilitates the hydrogenation of ethylene, but not benzene, in conformity with the multiplet theory and the considerations mentioned above.

The kinetics of hydrogenation from the point of view of the multiplet theory is considered in detail elsewhere ([13]).

There are grounds for assuming that some methods of activating and promoting catalysts are based on a change in the interatomic distances in the lattice. Highly interesting in this respect is the dependence [found by Rubinstein([14])] of the activity of different specimens of one and the same catalyst upon the magnitude of the lattice constant, which has a very distinct optimum.

From all that has been said it follows that the principle of conservation of the valence angle during the adsorption of a reacting molecule upon a doublet or, in the general case, on a multiplet, which is logically connected with the multiplet theory, is a highly important factor, and all results obtained on its basis are striking confirmation and further development of the cited theory of catalysis.

At the same time, as we saw above, a connexion is disclosed between the multiplet theory and the theory of the activated complex([5]) that appeared later. The multiplet theory provides models of the transition state, from which the theory of the activated complex may evaluate kinetics, the mechanisms provided by both methods being closely alike or completely identical, as, for example, in the case examined above.

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30. X. 1945.

[1] А. А. Баландин, ЖРФХО, **61**, 909 (1929); Усп. хим., **4**, 1004 (1935); ЖОХ, **12**, 337 (1942); Bull. Acad. Sci. URSS, sér. chim., **1**, 21, (1942); Z. phys. Chem., **2**, 289 (1929); **34**, 95 (1936). [2] J. H. Long, J. C. W. Frazer and E. Ott, J. Am. Chem. Soc., **56**, 1101 (1934). [3] P. H. Emmett and N. Skau, ibid., **65**, 1029 (1943). [4] М. С. Платонов, Тез. докл. конф. гет. катал., изд. АН СССР, 1940, стр. 35. [5] L. Pauling, The Nature of the Chemical Bond, N. Y., 1944, p. 409. [6] S. Glasstone, K. Laidler and H. Eyring, The Theory of Rate Processes, N. Y., 1941. [7] G. H. Twigg and E. K. Rideal, Trans. Farad. Soc., **36**, 533 (1940). [8] E. F. G. Herington, ibid., **37**, 361 (1941). [9] J. Horiuti and M. Polanyi, ibid., **30**, 663 (1934). [10] J. Horiuti, G. Ogden and M. Polanyi, ibid., **30**, 663 (1934); R. K. Greenhalgh and M. Polanyi, ibid., **35**, 511, 520 (1939); A. Farkas, L. Farkas and E. Rideal, Proc. Roy. Soc., A, **146**, 630 (1934); J. Am. Chem. Soc., **60**, 22 (1938); Trans. Farad. Soc., **33**, 827, 837 (1937); **35**, 906, 917 (1939); G. H. Twigg and E. K. Rideal, Proc. Roy. Soc., A, **171**, 55, 70 (1939); Trans. Farad. Soc., **35**, 934 (1929). [11] O. Beeck, A. E. Smith and A. Wheeler, Proc. Roy. Soc., A, 177, 62 (1940). [12] A. Sherman, C. E. Sun and H. Eyring, J. Chem.Phys., **3**, 49 (1934); G. Okamoto, J. Horiuti and K. Hirota, Sci. Papers Inst. Phys. Chem. Res. Tokyo, **29**, 223 (1936). [13] A. A. Balandin, Bull. Acad. Sci. URSS, sér. chim., No. 4, 339 (1945). [14] A. M. Rubinstein, ibid., No. 6, 427 (1943).

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PHYSICAL CHEMISTRY

ON THE INNER STRUCTURE OF A CUMULATIVE JET

By M. A. SADOVSKY

(Communicated by N. N. Semenov, Member of the Academy, 23. VII. 1945)

The experiments described here were set up with a view to measuring specific impulses reduced to unit area in coaxial sections of a cumulative jet at various distances from the charge, and to clearing up the question of the distribution of the impulses in the cross-section.

The measurements were made with a 270-kg ballistic pendulum of 150 cm effective length, fitted with a deep hollow steel cylinder open at one end to receive the cumulative jet.

An armour plate with an opening opposite the cylinder was set up between the pendulum and the charge of explosives. Into this opening diaphragms made of tool steel were inserted which cut out beams of desired cross-section from the cummulative jet.

The experiments were run with charges having a semi-spherical cumulative cavity 15 mm in radius. The diaphragms had openings 5, 10, 15 and 20 mm in diameter. The distance between the openings in the diaphragms and the charge was 20 mm in all the measurements.

The measured impulses were reduced to unit area of the diaphragm opening and compared with the reduced impulses from a standard equilibrium charge with a flat bottom, set off right on the cylinder fitted to the pendulum block.

In the standard tests the cylinder was equal in diameter to the charge. The results of the measurements are given in Table 1.

Table 1

Impulses S_k $\left(\frac{\text{kg·sec.}}{\text{cm}^2}\right)$ per Unit Area of the Diaphragm Opening (Distance, 20 mm; mean specific impulse, S_0*, of a standard charge, 0.176 $\frac{\text{kg. sec.}}{\text{cm}^2}$; explosive, amatol 80/20; density, 1.2)

Diameter of diaphragm, in mm	5	10	15	20
S_k	7.65	2.14	1.05	0.65
$S_k : S_0$	43.5	12.1	6.05	3.70

* The mean specific impulse, S_0, is the term we have applied to an impulse reduced to unit area of the butt end of the charge, without taking into account the distribution over the surface. Actually the specific impulse in the centre of a charge with a flat butt end is approximately double the mean impulse S_0.

In a series of tests the diaphragm opening was kept constant (15 mm in diameter) and the distance was varied. It was found that the impulse of a cumulative jet has a high density even at appreciable distances from the charge (Table 2).

Table 2

Change in the Impulse S_k $\left(\frac{\text{kg. sec.}}{\text{cm}^2}\right)$ of a Cumulative Jet as a Function of the Distance from the Diaphragm (Diameter of diaphragm opening, 15 mm; explosive, amatol 80/20, density, 1.2)

Distance to diaphragm, in mm	0.0	5.0	10.0	20.0	30.0	40.0	50.0	60.0
S_k	1.04	0.78	0.95	1.06	1.34	1.12	0.73	0.78
$S_k : S_0$	5.90	4.43	5.40	6.05	7.60	6.40	4.15	4.43

The results of the observations described above show that the density of impulse in a co-axial beam within a cumulative jet increases rapidly with a decrease in the diameter of the beam. Thus, a beam with a cross-section area equal to 1/36 of the cross-section of the cumulative cavity, contains up to 64 per cent of the momentum directed toward the bottom of the charge. Herewith the density of the impulse S_k in the beam is found to be 40 times greater than the density of the impulse S_0, observed during the explosion of a charge with a flat bottom. As the distance from the axis of the jet increases, the density of S_k decreases with extreme rapidity. If the impulse densities in concentric circular zones of the cross-section of the jet are computed from experimental data, it will appear that in the zone lying between $\frac{1}{6} r_k$ and $\frac{1}{3} r_k$ the impulse density is already about 30 times as low as in the centre. Table 3 gives some idea of the distribution of the impulse densities in concentric zones at various distances from the axis.

Table 3

Radii of circular zones expressed in terms of radii of the cumulative cavity	$\frac{1}{6} r_k$	$\frac{1}{6} r_k - \frac{1}{3} r_k$	$\frac{1}{3} r_k - \frac{1}{2} r_k$	$\frac{1}{2} r_k - \frac{2}{3} r_k$
Impulse in the circular zone.	7.65	0.272	0.194	0.124
S_k	43.5	1.55	1.10	0.70

The presence of a cumulative cavity practically does not affect the value of the total impulse, which is the same, within the limits of experimental accuracy, for both a cumulative and a standard charge. The influence of the cavity is confined to directing the jets of explosion products into a certain region where they all meet together, are pressed against one another and change direction to merge into a common stream, which retains at some length of its path the initial high density of matter and high rate of motion.

It seems probable that the increase of the impulse density in the jet is largely due to the increased concentration of explosion products, since the experiments involving determination of the jet velocity by methods requiring sizeable quantities of energy (such as the Dotrish method) and experiments involving the direct photography of the explosion of the charges with a cumulative cavity have failed to reveal any exceptionally large values of the velocity of the explosion products whatsoever.

It thus appears that the «velocity» conditions in the cumulative jet and on the surface of a normal charge, if not identical, are not very different. Therefore, the huge difference in the impulse density of these charges can only be explained by differences in the masses of the explosion products per unit of the affected area.

The observations carried out have left open the highly essential question as to the degree of concentration of the explosion products in the cumulative beam, since the raised values of the impulse may be explained as well by the impact of a highly concentrated mass of explosion products, as by the impact of the same mass of explosion products distributed in an appreciable volume extending along the direction of movement of the jet. The answer may be found either from measurements of the time the jet is in action, or from a direct determination of its density by methods of microsecond roentgenography.

It seems likely that there should be high concentration of the explosion products in the jet, and the distribution of the impulse densities along the radius of the jet, as revealed in the present paper, speaks in favour of this conjecture.

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GEOCHEMISTRY

ON THE GEOCHEMISTRY OF THE CARBONIFEROUS AND PERMIAN DEPOSITS OF THE CHISTOPOL REGION OF TATARIA

By A. B. RONOV, M. E. VLADIMIROVA, N. I. POLEVAYA and F. E. STARIK

(Communicated by V. G. Chlopin, Member of the Academy, 14. XI. 1944)

1. In 1941—1943 the Geochemical Laboratory of the Chistopol Oil-prospecting Expedition of the Radium Institute of the Academy of Sciences of the USSR carried on a systematic study of cores of Carboniferous and Permian rocks from wells drilled on the Zmiev, Malo-Tolkish and Buldyr uplifts situated in the territory of the Chistopol region of Tataria. The percentage content of CO_2 and SO_4 was determined in the specimens by the method of F. E. Starik. A total of over 1400 analyses were made, which made it possible to bear out in detail the quantitative relations of CO_2 and SO_4 in the lithologically «homogeneous» beds of the Middle Palaeozoic.

These data served as a basis for the cumulative diagram which, as may be seen from the figure, characterizes the section of the Palaeozoic, beginning from the lower part of the Tournaisian (C_1^t) up to the Kazan stage (P_2^{kaz}) inclusively. Plotted along the abscissae is the percentage content of CO_2 and SO_4; along the ordinates, the stratigraphic and lithological columns, absolute depths and points of sampling. The Tournaisian dolomitized limestones show an exceptional constance in the content of CO_2, the quantity of which varies within a range of 42 to 45 per cent. In this portion of the section there are practically no sulphates, and all the determinations give values below 1 per cent, *i. e.* values that escape measurement with the actual accuracy of the method employed (± 1 per cent).

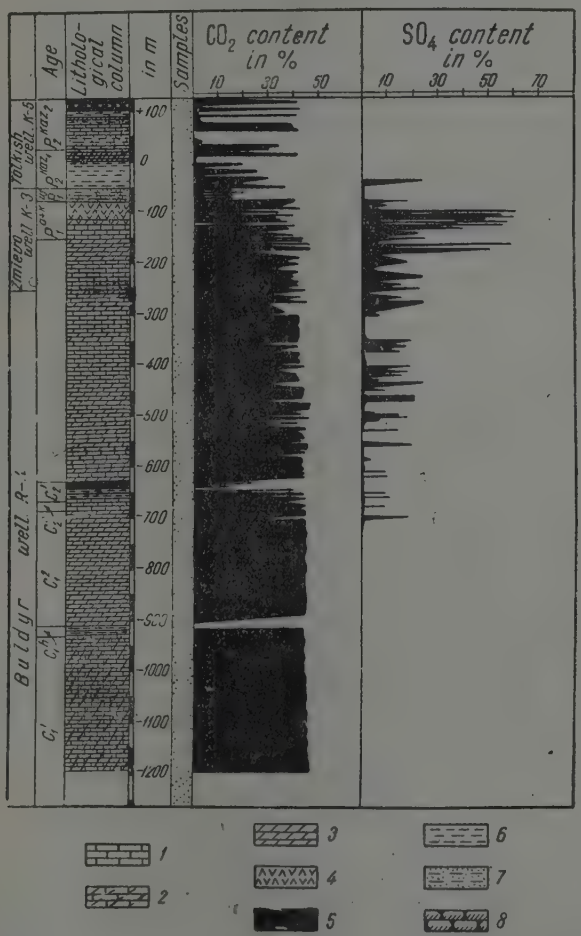
In the deposits of the coal-bearing series (C_1^{2b}) neither carbonates nor sulphates have been found, which the curve of the carbonate content illustrates by a deep depression.

In the section of Visean carbonate deposits the range of variation of the percentage content of CO_2 is somewhat wider than in the Tournaisian (from 38 to 45 per cent). The rocks contain no sulphates. The first appearance of the latter is confined to the topmost portion of the Visean (from a depth of —700 m), where the curve of the SO_4 content shows isolated, comparatively small peaks. Here, just as in all the superjacent horizons, corresponding to the peaks of SO_4 are depressions on the CO_2 curve. Thus, the one curve seems to be the mirror image of another one. The enantiomorphism of the curves is accounted for by the fact that they reflect the variations in the chemical composition of the two-component system of rocks. In those portions of the section where there appear considerable quantities of a hard component (terrigenous rocks), the diagram shows the conjugate curves to deviate from each other. Thus, in the case of the deposits of the Vereyan horizon (C_2) the two curves form depressions, against the background of which only small peaks of CO_2 (up to 40 per cent) and of SO_4 (up to 12 per cent) stand out, which are lithologically confined to limestone bands.

As we proceed upward within the section of the Middle (C_2^{2-5}) and Upper (C_3) Carboniferous, a marked tendency towards an increase in the sulphate content of

the rocks is recorded; this is accompanied by a corresponding decrease in the amount of carbonates. While the curve of sulphate content (see figure) shows an increase in the number and frequency of SO_4 peaks, the CO_2 curve correspondingly shows an increased number of minima.

A further enrichment of the rocks with sulphates continues in the deposits of the Lower Permian (P_1^{a+k}). The amount of SO_4 in this part of the section occasionally



Cumulative diagram of change in the carbonate and sulphate content of the Permian and Carboniferous rocks of the Chistopol region of Tataria. *1*, limestones; *2*, dolomitized limestones; *3*, dolomites; *4*, gypsum and anhydrites; *5*, marls; *6*, clays; *7*, clays with bands of sandstones; *8*, sandstones

reaches 60 per cent (gypsum and anhydrite bands). The curve of the sulphate content forms several high peaks, and near the roof of the Lower Permian carbonate-sulphate beds is abruptly broken downwards. The curve of the CO_2 content illustrates an inverse picture of development. The number of minima increases, pointing to a gradual impoverishment of the rocks in carbon dioxide.

In the deposits of the Ufa series (P_{1-2}^{uf}) and of the Kazan stage (P_2^{kaz}) sulphates occur only sporadically. The curves of the CO_2 content plotted for all the wells studied show a rhythmical alternation of maxima and minima of CO_2. Their num-

ber is the same everywhere, regardless of the fact that some of the wells are some 15—20 km apart. The curves of CO_2 content may doubtless be used as one of reliable methods for correlating the highly uniform series of the Ufa deposits.

2. From the analysis of the morphological peculiarities of the curves it may be seen that each of them expresses diametrically opposite laws of development. A study of the history of oscillating movements of the Earth's crust within the territory of the Volga-Ural region suggests the idea that the evolution of the chemistry of carbonate and sulphate rocks of the Carboniferous and Permian is closely associated with the general advance in the tectonic development of the region. The stable process of accumulation of the uniform carbonate series of Lower Carboniferous rocks coincides in time with the epoch of an intensive downwarping of the Earth's crust not only in the Chistopol region, but also throughout the entire Volga-Ural region (2).

A short but intense increase in the ascending movements of the Earth's crust during the Carboniferous time (C_1^{2b}) was reflected in the section in a universal development of the sand and clay terrigenous facies (a depression on the curve of the carbon content). Beginning with the close of the Visean age and during the whole of the Middle and Upper Carboniferous, the increase in the quantity of sulphates in the section proceeded along with an increase in the ascending movements in the Uralian geosyncline. Upon a shift of the geoanticlinal zone from east to west (1), in the same direction proceeded a shift of the facial zones of the Volga-Ural region.

Directly adjoining the geoanticline is a belt of terrigenous facies, and west of it proceeds an accumulation of the chemical components of erosion (2).

During the Artinskian age (P_1^2) the process of terrigenous decomposition of land (the western slope of the Urals), proceeding along with the intensification of the tendency of the uplift (3,5), supplies ever increasing amounts of sulphate solutions to the waters of the Volga-Ural epicontinental sea. During the Kungurian age (P_1^k) the ascending movements of the Earth's crust gradually extend to the parts of the Russian platform adjoining the Urals, and produce a shallowing of the basin. Lagoons are formed, the concentration of water increases, and as a result takes place the precipitation of hydrochemical sediments. On the curve of sulphate content this time is illustrated by a maximum of the percentage content of SO_4.

During the Ufa and Kazan ages a topographic approach of the zone of elevation to the Chistopol region resulted in that the region became a field of accumulation of mainly terrigenous deposits. The sulphates practically disappear altogether from the section, while in the formation of carbonate a rhythmicity, noted as far back as by M. E. Noinsky, is manifested, which reflects the rhythmical nature of the oscillatory movements of the Earth's crust of the second order. The zone of hydrochemical sedimentation at this time migrates south—to the region of Buguruslan, where in the spirifer deposits thick beds of gypsum, anhydrites and rock salt are developed.

The regular nature of the evolution of the chemistry of the sedimentary rocks of the Chistopol region is at places disguised by secondary processes associated mainly with an Akchagyl erosion, which had washed away the gypsum-anhydrite water-resistant horizon of the Lower Permian. As a result of this process the carbonate-sulphate series was leached out in a number of points of the region by surface waters, and converted to dolomitic flour.

The tectonic analysis applied by the authors in the study of the secondary processes within the territory of the region, made it possible to restore the original relations of the rocks, which proved to be analogous to those observed in the areas unaffected by the Akchagyl erosion (4).

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Received
14.XI.1944.

REFERENCES

[1] V. V. Beloussov, Bull. Acad. Sci. URSS, sér. géogr. et géophys., No. 4, 533 (1940). [2] В. В. Белоусов, Фации и мощности осадочных толщ Европейской части Союза, изд. АН СССР, 1945. [3] A. B. Ronov, Bull. Acad. Sci. URSS, sér. géogr. and géophys., No. 6 (1944), No. 2 (1945). [4] A. B. Ronov, C. R. Acad. Sci. URSS, XLVIII, No. 7 (1945). [5] A. B. Ronov, ibid., XLIV, No. 4 (1944).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 9

BIOCHEMISTRY

ON THE BIOCHEMICAL CHARACTERISTICS OF THE RESISTANCE OF PLANTS TO MICROORGANISMS

By B. A. RUBIN, O. I. PUSHKINSKAYA and V. E. SOKOLOVA

(Communicated by L. A. Orbeli, Member of the Academy, 16. VII. 1945)

The complex set of phenomena that are concomitant to infection in higher plants is both of theoretical and economic interest. The problem has consequently attracted the attention of a wide circle of students in the fields of physiology, phytopathology and biochemistry. The numerous studies which had been carried out within the last years went to demonstrate with ever increasing clearness that the resistance of plants is to be regarded as an active physiological process arising in them as a response to infection. The study of changes brought about in the plant's chemistry as a result of infection has shown that the degree of resistance of the host depends largely on certain peculiar features of its exchange, and, in the first place, on the complex of oxidation-reduction processes([6–8]).

The rôle of these processes is rather complicated and has not been fully deciphered. Of paramount importance is here the inhibitory effect produced by the oxidation processes([1]) upon the hydrolysing enzymes both in host and parasite. The rôle of oxidation processes in eliminating the deleterious effect of toxines is also of importance([2]). With increasing clearness is borne out the rôle played in the so-called phenolasic plants (after Szent-Györgyi's classification ([9,12,13])) by phenol transformations due to oxidation resulting from infection. These facts make it possible to assume the existence of a close relation between the resistance of the plant and its living conditions. And the existence of such a relation has actually been established in a number of studies ([10,11]).

In particular, interesting results have been obtained by Bunina([3]). It has been shown by this investigator that the resistance of the Kuban beet to moulds, reduced as compared with that grown in Georgia, is mainly due to the conditions of vegetation, the difference in resistance being observed under identical storage conditions. The intrinsic factors responsible for a different resistance of roots grown in contiguous geographical regions still remain unknown.

Observations made on commercial material, as well as special experiments carried out by us in 1941—1943 in Kirghizia and Kazakhstan, have shown that the local beet approaches the one produced in Georgia with respect to its resistance to a number of causative organisms. The high resistance of the Central Asia beet is manifest at any major stage characterizing the relations between the host and the parasite. In particular, the root-crops show a striking activation of the processes of the formation of new protective tissue (cutinization), which come to light as soon as the roots are injured mechanically, or upon artificial inoculation. In the latter case the areas of infection are rapidly localized through the formation of a kind of barriers on the way of infection, consisting of several layers of cells with cutinized walls. Aside from the resistance to the penetration and spread of

the infection, the Central Asia beets possess a genuine and very active physiological resistance.

These observations, as well as certain literary data made us suggest that among factors which affect the active resistance of the Central Asia beet, an important rôle should be ascribed to the altitude of the locality above sea level.

The experimental study of the problem was carried out in 1943 on material obtained by the Alma-Ata geophysical observatory from experiments on the social-

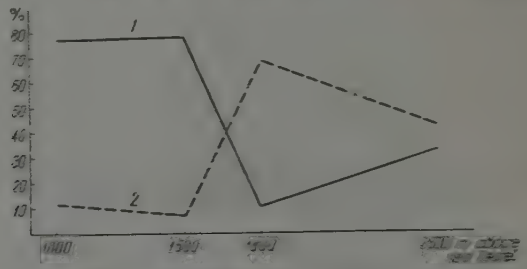


Fig. 1. Effect of altitude upon infection of roots in sugar-beet: *1*, heavy infection; *2*, slight infection

led vertical zonal distribution. Some plant material was kindly placed at our disposal by Vilenskaya, scientific worker of the observatory.

Sugar-beet was grown in the vicinity of Alma-Ata, at the following four places differing as to their altitude: 1) the Observatory, altitude 900—1000 m; 2) Medeo, 1500 m; 3) Ust-Gorelnik, 1800 m; 4) Verkhny Gorelnik, 2500 m.

The plants were harvested in October and subjected to investigation with respect to a number of biochemical indices, as well as with regard to their resistance.

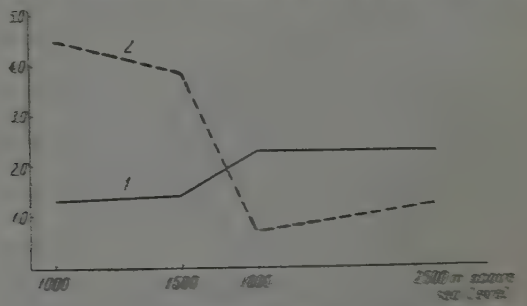


Fig. 2. *1*, relation of healing capacity to altitude (number of cells cutinized); *2*, proportion of rotten tissue as related to altitude

The resistance of roots was studied with the aid of artificial inoculation by the microbe complex comprising *Fusarium* sp., yeast (*Saccharomyces* and *Mycoderma*) sp. Prolonged preliminary observations have shown that this complex should be considered as the most active cause of the rot of the roots of beets, during vegetation and especially at the storage period.

The observations have shown that, alongside of the increasing altitude, the protective functions of the beet root grew, while its susceptibility to microorganisms decreased. This is brought out by the data on the proportion of roots which have been heavily or slightly injured by infection, as well as on the area of the injury (proportion of rotten tissue). One may see from the diagrams (Figs. 1 and 2) that beets from the altitude of 900—1000 m had 77 per cent of roots heavily infected, and 12 per cent slightly infected after a week following artificial inoculation.

The proportion of roots which have not shown any infection at all was 11 per cent only. In the material obtained from an altitude of 1800 m under similar conditions the relative number of roots heavily infected by rot was only one eighth of the previous figure. Above two thirds of the roots in this group proved either fully resistant or were infected but insignificantly. The data obtained from direct records of rotting fall in the same order.

The valuable properties of the beets grown at high elevation became especially conspicuous in the process of cutinization. It has been found from a large number of observations that in roots grown at an altitude of 1800 m the intensity of cutinization was nearly twice that in roots from an altitude of 900 m. A further increase in altitude up to 2500 m failed to increase the rate of cutinization. Among the factors of the plant's defence an important rôle is unquestionably played by cutinization.

Our three-year-long observations have shown that the roots of sugar-beet grown in Central Asia do not differ in any way from those grown in other regions of this country with regard to the character of their coatings. The conditions become enti-

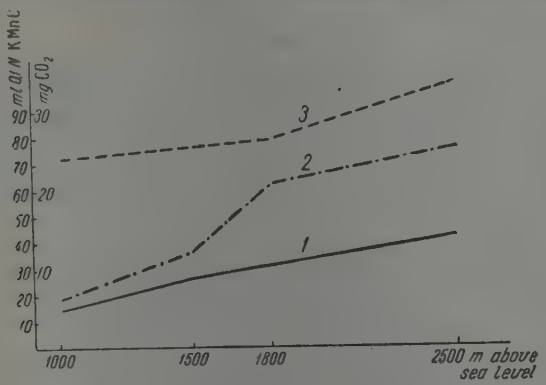


Fig. 3. Catalase activity (*1*) and peroxidase activity (*2*) (ml of 0.1 *N* $KMnO_4$) and respiration intensity (*3*) (mg of CO_2) of roots of sugar-beet grown at different altitudes

rely different when the external tissues of the root are damaged (cuts, wounds). The ensuing cutinization does not only assume a much more rapid course in roots from Central Asia, but also a larger number of cell layers of parenchyma are involved in it. The difference between the beets from Central Asia and those from Europe is thus due to their different ability to regenerate the damaged covering tissues.

As has recently been shown by Kursanov(5), healing of cuts in potato tubers is connected with an increased expenditure of oxygen on respiration. The same process of healing is intimately connected with the activation of peroxidase in the tissues of the tuber. The increased respiration and activation of peroxidase clearly show that the formation of suberine, which is the basis of cutinization of the cell walls, belongs to oxidation syntheses that require large amounts of energy. Thus, the increased cutinization at the points of cuts, which is characteristic of beets grown at great altitudes, undoubtedly reflects a higher level of oxidation-reduction processes peculiar to the tissues of the beet material.

This indirect evidence is supported by direct observations on catalase and peroxidase activity, as well as on the degree of oxygen respiration in the roots grown at different altitudes. Fig. 3 shows corresponding data from a series of observations.

From the curves one may see that along with elevation, there occurs a regular increase in peroxydase and catalase activity and in respiration intensity. These data fully coincide with observations of various writers who have reported an increase in vitamin C content, in oxidizing enzymic activity and in the total level of the oxidation processes in alpine plants (4).

The shifts in the chemical balance of sugar-beet observed at different altitudes are characterized in the first place by an intensification of the oxidation exchange. The intensification of oxidation processes is intimately linked with the resistance of beets, with the activation of the internal means of the plant's defence. This gives additional support to our previous findings as to the importance of oxidation processes for the phenomena of the resistance of vegetable organisms.

The results obtained in this study make it possible to approach the cause of the differences between the beet grown at Kuban province and in Georgia, which were mentioned at the outset of this note. It should be borne in mind that the beet grown in the Northern Caucasus is confined to a belt lying at an altitude of about 200 m, while in Georgia beet is cultivated at an elevation from 400 to 600 m. It is quite probable that the resistance of Georgian beets is due to the difference in the altitude of cultivation and to the modifications in the internal chemical processes due to it. Further investigation is in progress.

The A. N. Bach Institute of Biochemistry.
Academy of Sciences of the USSR.

Received
16.VII.1945.

REFERENCES

[1] Е. Арциховская, Биохимические основы устойчивости капусты к *Botrytis cinerea*, диссертация Ин-та биохимии АН СССР, 1941. [2] А. Бах. ЖРФХО, **42**, 2 (1912). [3] А. Бунина, Сборн. работ Биохим. сектора ЦИНС, **3**, 73 (1933). [4] С. Гребинский, Биохимия, **6**, 253 (1941). [5] А. Курсанов, Биохимия, **8**, 108 (1943). [6] А. Опарин и О. Купленская (Пушкинская), Тр. Майской сессии АН СССР, 60, 1935. [7] Б. Рубин, Биохимические основы хранения овощей, изд. АН СССР. 1939 и 1945. [8] Б. Рубин, Хранение сахарной свеклы, 1939. [9] H. Bahga u. K. Philippot, Z. f. physiol. Chemie, **258**, 147 (1939). [10] E. Gäumann, Rapp. du III Congr. intern. de pathol. comparée, **1**, 2, 39 (1936). [11] G. Jancke, Phytopathol. Z., **2**, 181 (1930). [12] K. Müller u. R. Griesinger, Angew. Botanik, 24, 130 1942). [13] A. Szent-Györgyi, Биохимия, **2**, 152 (1937); Bioch. Z., **233**, 236 (1931).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 9

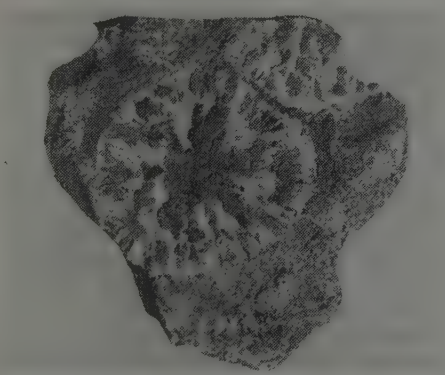
PETROGRAPHY

ON ORBICULAR GABBROS MET NEAR THE VOSKRESENKA VILLAGE, ARMENIAN SSR

By V. N. KOTLIAR

(Communicated by D. S. Beliankin, Member of the Academy, 3. VII. 1945)

In the course of a detailed study of the intrusives of the Little Caucasus the author encountered rather rare rocks known in literature by the name of orbicular gabbros. These rocks are located in the marginal part of a small intrusion of gabbro situated in Armenia near the Voskresenka village, 13 km east of the town of Kirovakan. The intrusion breaks through a volcanogene series of the Lower and Middle Eocene, being cut at one point by a dyke of diorites belonging to the Upper Eocene intrusive complex. Besides, the Voskresenka intrusion is essentially a westward extension of the Gokcha belt of ophiolites, which is known to be of Upper Eocene age.



Radially fibrous and zonal structure of the ovoids of the orbicular gabbro

The intrusion is somewhat stretched in the latitudinal direction and shows a rather complicated configuration. Its structure is no homogeneous one. It is largely composed of a coarse-grained gabbro consisting of plagioclase (bytownite—anorthite), monoclinic pyroxene, hornblende and small amounts of hypersthene, biotite and of less frequent olivine. Less widespead is a fine-grained and a porphyraceous variety of gabbro of a similar mineralogical composition. The orbicular gabbros have been reported from the north-eastern and south-western peripheral parts of the intrusion, where they form a belt up to 15 m wide. In certain places the orbicular gabbros are alternating with banded gabbros.

A study of the relations existing between the separate varieties showed that the orbicular gabbros were formed upon the intrusion of coarse-grained, but previous to the intrusion of fine-grained gabbros. The orbicular gabbros form melanocratic rocks with large orbicular ovoids from 2 to 18 cm (on the average, about 4 cm) across. The structure of the ovoids is radially-fibrous (see figure), frequently zonal. Their central part is usually represented by crystals of hornblende or monoclinic pyroxene, and less frequently, by small xenoliths. The central zone is usually composed of light-coloured crystals of anorthite and hornblende: and the outer zone, of smaller intergrowths of hornblende and bytownite.

Of less frequent occurrence are orbicular gabbros with light-coloured ovoids, consisting mainly of plagioclase; still less frequent are gabbros with ovoids consisting entirely of hornblende or multizonal, in which the core consists of hornblende. Further follows a zone of radially arranged crystals of anorthite and hornblende, then comes the zone of a fine-grained aggregate of plagioclase and hornblende and, finally, the periphery, consisting of radial intergrowths of plagioclase and hornblende. In the surrounding mass epidote and prehnite segregations are sometimes noted. Under the microscope a radially stretched arrangement of the crystals is well pronounced, which is the best developed in the central zone.

The surrounding mass of the orbicular gabbro has a gabbro structure and consists of hornblende, monoclinic pyroxene, bytownite, as well as of sphene and magnetite. At places the contacts with the ovoids consist entirely of hornblende, completely replacing pyroxene. A very close resemblance in the composition between the mass enclosing the ovoids and that of the central part of the latter is characteristic. Orbicular rocks have been so far described mainly from California (San-Diego), from Corsica (Santa Lucia) and from Central Finland (Kangasniemi).

The origin of orbicular textures was discussed by a number of eminent petrographers. Loewinson-Lessing and Vorobieva [1] considered these textures as a result of autobrecciating upon a repeated intrusion of magma. Sederholm [2] explained their formation by the crystallization of viscous magma and a successive segregation of crystalls around cores: and the zoning, by a fractional differentiation during a slow diffusion. Escola [4], rejecting an interaction between the inclusions and the magmas, explained the formation of these textures by metamorphic differentiation and metasomatic replacement in the parent magma. Beliankin and Petrov [3,5], who established for the orbicular granites of Kangasniemi a similarity in the composition of the core of the ovoids and of that of the surrounding mass, explain their formation by intense contact phenomena under regeneration of granite induced by gaseous emanations, the different elements of granite varying as to their fusibility.

In the case here considered it is necessary also to take into account the influence of the contact which resulted both in mechanical decomposition of the enclosing rocks and in an interaction between them and the intruding magma. The formation of concentric zones, particularly of zonal ovoids, may be accounted for not only by the influence of gaseous emanations, but by a concretional crystallization as well.

Received
3.VII.1945.

REFERENCES

[1] F. J. Loewinson-Lessing and O. A. Vorobieva, C. R. Acad. Sci. URSS (1929). [2] Д. С. Белянкин и В. П. Петров, Тр. Петр. ин-та АН, вып. 3 (1932). [3] J. J. Sederholm, Bull. Commiss. Geol. de Finland, No. 83 (1928). [4] P. Escola, J. of Geol., XLVI, No. 3 (1938). [5] D. S. Beliankin and V. P. Petrov, J. of Geol., XLVII, No. 7 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 9

PALAEONTOLOGY

ON A POORLY STUDIED BUT IMPORTANT GROUP OF FOSSIL ORGANISMS

By A. G. VOLOGDIN, Corresponding Member of the Academy, and C. B. CORDET

The usual objects of palaeontological study are remains of fossil animals, or imprints of plants, which are mostly characterized by rather large dimensions. The remains of the organisms studied are therefore often termed the «macrofauna», as distinguished from the microfauna, to which are referred the remains of the foraminifers. On the other hand, literature often contains references to smaller organic remains, microscopic algae and bacteria, the geological and stratigraphical rôle of which in sedimentary deposits still remains obscure.

One of the earliest references to the presence of bacterial remains in Palaeozoic beds was made by van Tieghem(1). A number of papers by Renault dealt with the same subject: within the period between 1895(2) and 1900 he discovered remains of saprophytic bacteria in beds of different geological age, beginning with the Devonian. Walcott thought it probable that bacteria might play an important part in the formation of carbonate rocks. In one of his papers he gives photomicrographs of stained coccus-like corpuscles of bacteria ($\times 1100$), observed in the form of sparse chains from Algonkian beds of the Gallatin Formation, Montana(3). This and similar findings are mentioned by Moodie(4). The exceptionally bright coloration of the micrococci of Walcott, however, arouses some doubt as to their Algonkian origin. They do rather remind recent bacteria that occasionally got on to the object-glass.

In this country the problem of fossil bacteria was dealt with by Zalessky, who observed bacteria-like bodies, «point-micrococci», in beds of oil shales of the Kuker series of the Silurian of the near-Baltic area(5). This author discovered remains of microorganisms also in Lower Volga oil shales of Upper Jurassic age, in the composition of which blue-green algae took part(6). Tausson attributes to bacteria of the saprophytic group an important rôle in the transformation of fossil organic matter to different kinds of fossil coals(7). Like many other investigators, he has often noted evidence of the destructive activity of bacteria in humic coals, resembling the results of the activity of recent microbes. Tausson is therefore inclined to consider the bacteria-like corpuscles observed by many authors in rocks as bacteria which at remote geological epochs were already represented by different groups and were taking part in the decomposition of organic matter. He presumes that microorganisms could also participate in the formation of iron and sulphur ores, limestones, *etc.*

The questions of the life-activity of living sulphur, iron, hydrocarbon and other bacteria were considered in detail by B. L. Isachenko, L. D. Sturm, N. G. Cholodny and others, who had frequently occasion to observe the specific nature of the microbial activity under laboratory conditions.

The problem of fossil bacteria, fossilized bodies of bacteria of past geological epochs and of the possibility of their fossilization itself, particularly of autotrophic bacteria, remained obscure.

While engaged in the study of remains of the oldest carbonate algae of the group of blue-green algae, the authors were largely interested in the problem of fossil bacteria, since the existence of the recent *Cyanophyceae* is impossible without the participation of corresponding bacterial coenoses.

It seemed necessary to investigate the algal limestones and dolomites of the pre-Cambrian and Palaeozoic for the content of bacterial remains in order to shed light upon the rôle of bacteria in the development of algae, in processes of lithogenesis, in the accumulation of certain biogenic deposits in the sediment on the bottom of the ancient basin.

Having at their disposal a rich collection of sedimentary rocks of different geological age, some of which were either undoubtedly or presumably biogenic, the present authors made the above-mentioned attempt. It implied the use of especially thin transparent sections; sometimes it proved also necessary to stain and to view them under the microscope with a magnification unusual for lithologists and palaeontologists, up to 1800—2000 times. The effect obtained was remarkable. The present paper contains a brief preliminary report on the first results obtained.

The new method for the study of sedimentary and biogenic rocks was primarily applied to algal rocks from Permian deposits of the Shchugor River (basin of the Pechora, collection of A. A. Chernov), where oncoliths were macroscopically established. According to data of chemical analysis, algal rocks were distinguished by an increased content of iron and manganese. Potassium ferrocyanide in the presence of hydrochloric acid showed a blue coloration of iron compounds. A thin-section prepared from this rock showed that microorganisms played a very important rôle in its formation. In the field of vision of the microscope a large accumulation of microcorpuscles was observed, clearly resembling bacteria. The membranes of these corpuscles, which were undoubtedly bacterial cells, were dark from iron oxides; this made it possible to see, without any artificial staining of the thin-section, rods stretched in filaments, chains, closely and fancifully interlaced, with intervals occasionally not exceeding 1.2 μ. The diameter of the bacterial cell was 1.3—1.8 μ against a length of 1.9—2.33 μ. The bent of the filaments corresponded to the points of articulation of two adjacent rods. It seemed as if along with microalgae, through whose activity these oncoliths were formed, colonies of iron-bacteria were intensively developing, morphologically very well preserved in the fossil state, as it seems, owing to the presence of the residual organic matter of bacterial cells.

It seemed of great interest to inquire into the nature of the ferruginous cement of the sandstones which is known to occur frequently. A thin-section from such a rock (from the collection of A. A. Chernov from the Shchugor River in the basin of the Pechora River) revealed an amazing micropicture. The whole space between the particles of the clastic material was found to be filled with ferruginous membranes of bacilli and their chains of fossil iron-bacteria. It should be noted that when preparing thin-section, the micromaterial of the cement together with Canada balsam becomes often spread over the surface of the sections of quartz or other grains, so that remains of iron-bacteria seem to be present inside of these grains, too. It is easy to demonstrate that this impression is misleading. The ferruginous sandstones, in which the cement consists of iron oxides, are derived from different sands that had originally been saturated with waters of a definite degree of mineralization and containing bacteria. Whether the iron brought by the water circulating in the sands was assimilated, or extracted from adjoining ferriferous mineral particles, is not clear as yet. Both processes have probably been operative here.

It is a matter for regret that the process of diagenesis of friable sediment in a solid rock has up to the present time been given but very little attention on the part of investigators. While the essence of this process is still a subject of controversy, its chemical, mineral nature seems to leave no room for doubt. The erroneous underestimation of the possible rôle of microorganisms in the transformation of mineral sediments is now evident to us. A purely «chemical» approach to the

process of diagenesis of rocks, as attempted by Pustovalov, is obviously insufficient to settle this question (8).

To microscopic study were subjected many specimens of rocks of the Lower Palaeozoic from Kazakhstan and Western and Eastern Siberia. Among the Kazakhstan material especially interesting proved to be rocks of the phosphorite-bearing series of the lower part of the Middle Cambrian of Kara-tau.

Investigated were also phosphorites of an oölitic habit, externally resembling bauxite ores, for which they were originally mistaken by some geologists. Such a rock with grey oölitic grains and yellow-brownish cement, when acted upon by potassium ferrocyanide, gives in the presence of hydrochloric acid a positive indication of iron. The oölitic grains up to 4—5 mm across were found to be the place of abundant accumulations of microcorpuscles of coccus-like bacterial cells (1.1 μ in diameter and 1.33 μ in length), arranged in filaments, chains or singly. In a thin-section requiring no artificial staining one is able to distinguish tightly packed microcorpuscles, variously oriented in space, which can readily be identified as bacterial cells with excellently preserved outlines, whose activity was to transform phosphorus compounds.

It is of interest to note that inside the oölitic grains the arrangement of microcorpuscles is a more disorderly one than at the periphery, where a trend towards a concentric habit of the spheroids is manifest, and where the bacilli tend towards radial orientation. At places to the temporary surfaces of the concentric layers correspond accumulations of other bacteria which, by their dimensions and the yellow-brown colour of the rock, are readily identified as bacilli and filaments of ferro-bacteria. The cement of the oölitic phosphorous ore, which shows a similar yellow-brown colour, consists entirely of fossilized cells of iron-bacteria; this should be taken to mean that the processes of phosphorus and iron accumulation took place at different times, the primary accumulation of the pea-like phosphate sediment under indubitable participation of bacteria, and the cementation of this sediment into a solid rock, at the stage of diagenesis by iron bacteria.

By clearing up the nature of the Kara-tau phosphorites which are found to be an indubitable microbiogenic sediment of the Cambrian sea, we settle the ancient controversy in which the chemical standpoint often prevailed over other hypotheses. Further research work in this field based, upon the new results obtained from the study of the Kara-tau phosphorites is likely to shed light upon the rôle played by microorganisms in the concentration of phosphorus, iron and manganese.

As well known, phosphorites make up thick beds at Kara-tau. Their total thickness occasionally attains 20 to 30 m; in the north-eastern Kara-tau phosphorite-bearing series have been traced over 120 km, which is evidence of the important rôle played by microorganisms in the formation of rocks and accumulation of phosphorus.

In 1944 one of the authors visited the manganese mines of Siberia—the Ussa (in the Kuznetsk Ala-tau) and the Mazulka (in the Arga Range, near the town of Achinsk)—associated with Lower Palaeozoic—Cambrian deposits. The sedimentary origin of the manganese ores on the Ussa River had never been questioned, although the facies of manganese accumulation was rather obscure. As to the Mazulka deposit, it was for a long time considered now metatectic now hydrothermal. The recent discovery of primary manganese and iron ores, such as rhodonites, rhodochrosites, siderites, in the lower part of the ore-bearing series represents the problem as to the origin of these ores under a new aspect.

It is of interest to note that the manganiferous rocks had in both regions been undoubtedly formed under the participation of manganese and iron-bacteria, the corpuscles of which, undistinguishable in the solid rock, are readily identified in thin-sections even of poorer ores if high magnification is used. The elementary microcorpuscles-bacilli are 1.33 μ across and 2.0—2.6 μ long; they are usually connected in dark interwoven filaments.

Interesting enough, the secondary oxidized manganese ores of the Mazulka mine, that occur as numerous irregularly shaped bodies above the zone of primary ores and belong to a later period (possibly to the Mesozoic), are entirely composed

of sheaths of manganese bacteria in the shape of bacterial cells about 1.4 μ in diameter, with a length of 2.21—25 μ, linked into fancifully curved, closely spaced filaments.

Of no less interest is the fact that the authors have established a continuous profuse accumulation of sheaths of iron-bacteria also in the ferruginous shales of the Grebenka series of the Proterozoic of the Angara River.

Being but the first results of a revision of sedimentary deposits viewed at a special angle, these new data, bearing upon the distribution of remains of autotrophic bacteria in Proteozoic and Palaeozoic strata, seem to be highly promising. At the time being it is obvious that the rôle of microbiological processes in the formation of rocks, as well as in their destruction, is exceedingly important. As to their concentrating work with regard to definite chemical elements and compounds, it is not only of scientific interest, but also of extreme economic importance.

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REFERENCES

[1] Cl. Reid, Art. Paleobotany in Encycl. Britan., 11th ed., p. 525, 1911. [2] B. Renault, Bull. de Mus. d'Hist. Nat., par. 1—4, 168. [3] C. D. Walcott, Proc. Nat. Acad. Sci, 256 (1915). [4] R. L. Moodie, Paleopathology, III, 1923. [5] M. D. Zalessky, Bull. Acad. Sci. URSS, XI (1917). [6] М. Д. Залесский, Изв. Сапр. ком., в. 4 (1928). [7] В. О. Таусон, Арх. биол. наук, XLIII, в. 2—3 (1936). [8] Л. В. Пустовалов, Петрография осадочных пород, ч. II, 1940.

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MICROBIOLOGY

FIXATION OF ATMOSPHERE NITROGEN BY *AZOTOBACTER* AS INFLUENCED BY THE SUBSTANCES REACTING WITH THE AMINO-GROUP OF THE CATALYST

By M. V. FEDOROV

(Communicated by A. A. Richter, Member of the Academy, 6. VIII. 1945)

Aldehydes and nitrites hold a place of their own among other enzymic poisons. Although different in their chemical structure, they react with the amino-group of the catalyst, reducing the rate of the reaction catalysed by the latter. But along with these common peculiarities, they also differ in certain points. The nitrites inactivate such enzymes as contain the amino-group mainly in an acid medium, with a pH below 5. The aldehydes also inactivate the catalyst, but only in the case where the latter does not react with its substratum through the amino-group. The inhibitory effect of the aldehydes becomes also manifest in oxidation processes connected with the structural surface of cells (*e. g.* in *Bacterium coli*).

In our preceding investigations we have shown[2,3] that the process of fixation of atmosphere nitrogen by *Azotobacter* is connected with the catalysing surface; it seemed therefore of interest to find out, whether the amino-groups are constituents of this catalyst and whether they react with the substratum catalysed by them.

With this purpose in view we carried out experiments on the influence of sodium nitrite, and also of acetic and glyceric aldehydes upon the fixation of atmosphere nitrogen. The two last substances were chosen because they are readily assimilated by *Azotobacter* in the case their aldehyde group is oxidized to the carboxyl group (acetic and glyceric acids), or reduced to the alcohol group (ethylic alcohol and glycerine).

All the experiments were carried out in Erlenmeyer flasks 300 ml in volume. 100 ml of nutrient medium for *Azotobacter* (in our modification) containing 1.8 g of inverted sugar were put into each flask. The pH of the medium was about 7. After sterilization the medium was inoculated with 1 ml of *Azotobacter agile* suspension (the film from a test-tube with slant agar was shaken up with 100 ml of sterile tap water), and then kept in a thermostat at a temperature of 30° C during 14—16 days. At the end of each experiment the medium was tested for sugar content after Bertrand (upon the removal of acetic aldehyde), and the total nitrogen content after Kjeldahl (in experiments with $NaNO_2$ in Iodlbauer's modification). In addition, in each experiment the number of *Azotobacter* cells grown were directly counted under the microscope.

The first experiment was carried out with acetic aldehyde added up to the concentration of 0.001, 0.01, 0.05 and 0.1 mol. The results of the experiment are given in Table 1.

The data of Table 1 show that the effect of acetic aldehyde is strongly to inhibit not only respiration, but fixation of atmosphere nitrogen, too. When the concentration of acetic aldehyde was equal to 0.05 mol., sugar oxidation and intensity

Table

Influence of Acetic Aldehyde upon Fixation of Nitrogen by *Azotobacter*

Concentration of acetic aldehyde in the medium, in mol.	Amount of sugar assimilated, in g	Intensity of sugar assimilation, in % to control	Amount of nitrogen fixed in mgs			Intensity of nitrogen fixation, in % to control	Number of cells of *Azotobacter* grown
			per 1 g of sugar in individual culture	per 1 g of sugar on the average	per 1 g-mol of hexoses		
Control (without acetic aldehyde)	1.80 1.80	100.0	8.20 8.67	8.44	1519.2	100.0	2.4×10^{10}
0.001	1.66 1.67	92.5	8.81 8.80	8.80	1584.0	104.2	2.76×10^{10}
0.01	0.94 0.98	53.3	3.73 3.43	3.58	644.4	42.4	2.88×10^{10}
0.05	0 65 0.45	30.6	0.51 0.70	0.61	109.8	7.2	0.8×10^{10}
0.10	0.25 0.16	11.4	0.00 0.00	0.00	0.0	0.0	—

of nitrogen fixation were not higher than 30 and 7 per cent to the control, respectively. The fact that the both processes mentioned are found to be suppressed to a different degree by the same dose of acetic aldehyde should be taken to mean that they take place on different catalysing surfaces.

Special attention belongs to the fact that when the aldehyde concentration is equal to 0.05 mol., nitrogen fixation is almost entirely suppressed. This fact is to be interpreted in the sense that at a concentration mentioned the aldehyde group

Table 2

Influence of Glyceric Aldehyde upon Fixation of Nitrogen by *Azotobacter*

Concentration of glyceric aldehyde in the medium, in mol.	Amount of sugar assimilated, in g	Intensity of sugar assimilation, in % to control	Amount of nitrogen fixed, in mgs			Intensity of nitrogen fixation, in % to control	Number of cells of *Azotobacter* grown
			per 1 g of sugar, in individual culture	per 1 g of sugar on the average	per 1 g-mol of hexoses		
Control (without glyceric aldehyde)	1.71 1.80	100.0	8.11 8.27	8.19	1474.2	100.0	3.566×10^{10}
0.001	1.65 1.65	94.0	8.48 8.61	8.54	1537.2	104.3	2.486×10^{10}
0.01	1.60 1.60	91.0	8.54 8.10	8.32	1497.6	101.6	3.442×10^{10}
0.10	0.55 0.77	37.6	0.51 0.45	0.48	86.4	5.8	1.266×10^{10}

of this substance completely fixes the amino-group of the catalyst, thus eliminating the latter from the reaction.

The fact that the catalyst of nitrogen fixation is completely inactivated by aceto-aldehyde should be interpreted to mean that the latter does not only contain the amino-group, but that the amino-group and the reacting substratum do not interact with each other either.

The second experiment was performed with the glyceric aldehyde, the concentration of the latter being equal to 0.001, 0.01 and 0.1 mol. The results of the experiment are given in Table 2.

The influence of the glyceric aldehyde was found to be quite similar to that of acetic aldehyde. Low doses of this substance (from 0.001 to 0.01 mol.) produce scarcely any influence upon the intensity of sugar assimilation and fixation of atmosphere nitrogen, while higher doses (0.1 mol.) strongly reduce both the intensity of sugar oxidation and the intensity of nitrogen fixation. At the concentration mentioned the intensity of fixation of atmosphere nitrogen is equal only to 5.8 per cent to the control.

The only difference in the action of the substances compared is that a similar degree of inhibition of the processes considered is due to the different concentration of acetic or glyceric aldehydes, *viz.* about 0.05 mol. and 0.1 mol., respectively. The most probable explanation of this differential influence is that their rate of penetration into the cells is different.

Similar results were obtained in experiments with sodium nitrite, added to the nutrient medium to the concentration of 0.0001, 0.001, 0.005 and 0.01 mol., respectively. The results are given in Table 3.

Table 3

Influence of Sodium Nitrite upon Fixation of Nitrogen by *Azotobacter*

Concentration of sodium nitrite in the medium, in mol.	Amount of sugar assimilated, in g	Intensity of sugar assimilation, in % to control	Amount of nitrogen fixed in mgs			Intensity of nitrogen fixation, in % to control	Number of cells of *Azotobacter* grown
			per 1 g of sugar in individual culture	per 1 g of sugar on the average	per 1 g-mol of hexoses		
Control (without $NaNO_2$)	1.67 1.58	100.0	8.26 8.11	8.18	1472.4	100.0	6.0×10^{10}
0.0001 (0.14 mg N) . . .	1.48 1.58	94.2	10.09 10.19	10.14	1825.2	124.0	2.99×10^{10}
0.001 (1.4 mg N) . . .	1.49 1.53	92.9	8.22 8.74	8.48	1526.4	103.7	2.34×10^{10}
0.01 (14 mg N) . . .	0.95 0.95	58.5	0.00 0.00	0.00	0.0	0.0	4.97×10^{10}
Control+10 mg N—NH_4Cl	1.56 1.63	98.2	4.09 3.67	3.88	698.4	47.4	2.17×10^{10}

The influence of nitrite was found to differ from that of aldehydes in that small doses of nitrite (0.0001 mol.) clearly stimulated the process of nitrogen fixation, while large loses (0.01 mol.) completely suppressed this process. However, there was no positive correlation between the inhibition of nitrogen fixation and the failure of *Azotobacter* to develop. As a matter of fact, the relations observed were rather negative. When the concentration of sodium nitrite was high, an intense

growth of *Azotobacter* was observed, while the fixation of atmosphere nitrogen was entirely inhibited.

Apparently the living cells had at their disposal an amount of compounds containing the fixed nitrogen sufficient for a normal metabolism and synthesis of protein compounds of the protoplasm. Besides, as it has justly been pointed out by Stoklasa ([4]) and Kostychev ([1]), the oxidized form of nitrite nitrogen is previously reduced to ammonia. *Azotobacter* cells are multiplying very intensely even when the concentration of sodium nitrite in the medium equals 0.01 mol.; during the same period 4.97×10^{10} cells were found to develop in 100 ml of the medium. On the other hand, the fixation of atmosphere nitrogen is suppressed at a concentration of nitrite in the medium as low as 0.005 mol.

Two ways may be suggested when looking for the cause of the phenomenon described: 1) nitrites inactivate the catalyst through the decomposition of its amino-group; 2) they substitute the molecular nitrogen by accessible forms of fixed nitrogen. The first explanation seems to us more plausible not only because of the complete suppression of nitrogen fixation brought about by the fixation of the amino-group of catalyst by aldehydes, but also because of the considerable intensity of nitrogen fixation that continues when ammonia salts, instead of nitrites, are introduced into the medium. The data of Table 3 show that when 10 mg of ammonium nitrogen are present in the medium, the intensity of nitrogen fixation reaches 47.4 per cent to the control.

It seems quite evident that the elimination of the amino-group of the catalyst from the sphere of catalysis is responsible for the suppression of the reaction. A still stronger inhibition should be expected under the influence of nitrites if the reaction of the medium were more acid (pH = 5), but experiments of this kind are impracticable, since *Azotobacter* fails to grow under these conditions.

The results of the experiments briefly described above are of great interest. They show that the amino-group is a constituent of the catalyst of nitrogen fixation. They may, therefore, be considered as a first step in finding out the chemical structure of the catalyst itself, without isolating it from the living cells of *Azotobacter*. It is true, however, that the problem as to the rôle of the amino-group in catalysis is not yet definitely solved. The inactivation of the catalyst by aldehydes only shows that it reacts with the substratum catalysed otherwise than through the amino-group.

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Received
6.VII.1945.

REFERENCES

[1] С. П. Костычев, А. Рыскальчук и О. Швецова, Тр. отд. с.-х. микробиологии ТОА, I, 91 (1926). [2] М. В. Федоров, Микробиология, XIV, в. 2 (1945). [3] М. В. Федоров, Микробиология, XIV, в. 3 (1945). [4] J. Stoklasa, Ber. d. Dtsch. Bot. Ges., **24**, 22 (1906).

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PLANT PHYSIOLOGY

A CONTRIBUTION TO THE STUDY OF THE FUNCTION OF THE CHLOROPHYLL-BEARING TISSUE IN THE CONDUCTIVE BUNDLES OF THE STEM IN *CUCURBITA PEPO*

By M. MOISSEJEWA

(Communicated by A. A. Richter, Member of the Academy, 5. VIII. 1945)

While investigating stems of *Cucurbita Pepo* on fresh material I happened to notice that the distribution of the chlorophyll-bearing tissues varied within the stems with the age of the internodes(1). In young internodes chloroplasts were present in all the tissues except for the epidermis and vessels, while in old internodes green plastids were contained exclusively in the living cells of the vascular bundles, *viz.* in phloem, cambium and in wood parenchyme where they remained green and active until late in autumn. Knowing that growth substances are formed in the green parts of the plant, one might suggest that the chloroplasts present in the bundles of the stem also take part in the formation of these substances. This suggestion found some support in the study of the callus which is formed when stem-cuttings are made.

There are indications that «in the presence of other additional substances the auxine seems to cause elements of certain tissues that are always the same to pass into embryonic condition and into division in connexion with regeneration processes»(2). Only living elements of the vascular bundles, those possessing green plastids, take part in callus formation(1).

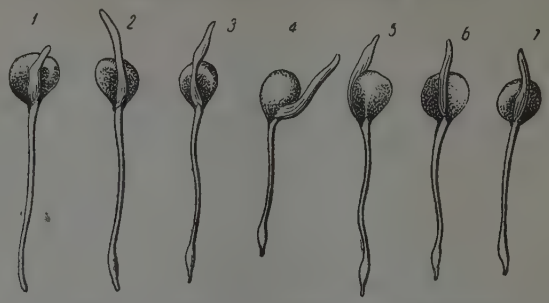
In order to check up the suggestion as to the presence of growth substances in the chlorophyll-bearing tissue of the vascular bundle corresponding experiments were necessary. The more so in view of the fact that Meyer(2), who studied by the «paste method» the distribution of the growth substances in *Cucurbita Pepo* throughout the period of development of the plant, has found that neither a six-day-old stem nor portions of a three-month-old stem contained any growth substances.

In my experiment with roots of Indian corn I made use of Cholodny's method, previously tested in experiments on *Azotobacter* (4). When three tips of coleoptiles of *Zea* are placed on the root tip of the same species, the longitudinal growth of the root is greatly reduced under the influence of the auxine secreted by the coleoptiles; within 16—20 hours a tumour-like thickening is formed on the root tip. According to Cholodny, a more rapid differentiation of the tissues takes place in the meristematic region of the root under the influence of the additional hormone and in connexion with the thickening (3).

The experiments were carried out in August, 1944, with seedlings of Indian corn (mainly of the variety «Zhemchuzhinka») grown in saw-dust, and on stems of *Cucurbita Pepo* (lower and middle internodes).

Plantlets of Indian corn possessing 1—3 cm long rootlets were cautiously removed from the saw-dust, rinsed in water and kept in water for some 15—20 minu-

tes. Three small fragments of tissue, about $1 \times 3 \times 1.5$ mm³*, were superimposed on the meristematic region of the root from three sides, with their long sides along the root, and with their transverse sections, towards the root. The plantlets were



1, control; *2—3*, after a 14-hour treatment with three tips of coleoptiles of corn; *4—7*, after treatment during the same period with three pieces of vascular bundles of pumpkin

kept in vertical position for 12—17 hours in a room saturated with moisture, at a temperature of 21—24°.

It should be noted that every 2 or 3 hours throughout the course of the experiment the roots had to be sprinkled with water; otherwise, because of the shortage of water, they would have given a reduced growth, so that the difference between the control and the experimental roots would be obscured.

To obtain a more precise idea of the amount of the hormone in the tissue tested, parallel experiments were arranged with tips of coleoptiles of Indian corn.

The experiment has shown that the best results were obtained when the longitudinal strips of the tissue of each kind were kept for several minutes separately from other tissues in water, and then cut into portions of a desired size and applied to the root.

Parts of plants or tissues applied to the root tip	Number of roots in experiment	Number of roots in control	Average increment of experimental roots, mm	Average increment of control roots, mm	Difference, mm	Difference, %
3 tips of coleoptiles of *Zea*	20	24	5.9±0.5	17.3±0.8	−11.4	−65.9
Complete bundle from main stem or from large side branches of outer or inner ring	32	20	7.9±0.5	16.0±0.9	− 8.1	−50.6
Xylem	10	10	8.4±0.6	16.1±1.6	− 7.7	−47.8
Phloem, inner and outer	22	12	9.8±0.3	15.5±0.7	− 5.7	−36.7
Cambium	14	12	10.0±0.5	17.1±1.2	− 7.1	−41.4
Ground tissue of stem	12	11	16.6±0.9	18.9±0.9	− 2.3	−12.1
Vascular bundle of fruit-stalk	13	8	7.5±0.5	13.3±1.3	− 5.8	−43.6
Vascular bundle of petiole	16	8	10.6±1.0	19.7±1.3	− 9.1	−46.1
Pollen of pumpkin	8	10	5.1±0.5	19.7±1.2	−14.6	−74.1

As may be observed from the table, the vascular bundles of pumpkin contain a very large amount of growth substances. The whole vascular bundle of the stem and the elements composing it—phloem, cambium and xylem—both caused a strong

* Only in experiments with cambium was it necessary to cut out very narrow strips in order not to involve phloem and xylem, and to use 4—6 pieces, instead of 3, applying them with their transverse sections towards the root. These small fragments were sometimes turned towards the root with their tangential surface, while sprinkled.

decrease in the rate of growth of the root, although to a somewhat lesser degree than 3 tips of the coleoptiles of corn, at the same time they produced a thickening at the end (see figure).

The vascular bundles of the fruit-stalk and of the petiole also strongly checked the growth of the root and produced tumours (see table). The ground tissue of the stem (and of the fruit-stalk) contains a relatively small amount of growth substances; under experimental conditions it did never produce any thickening at the root-tip, but always checked the growth to a rather considerable measure.

It should be added that the pollen of pumpkin was found to check the growth of the root even stronger than the coleoptiles of corn (see table), the tumour produced being shorter and thicker than in other instances.

On the ground of the data presented above we arrive at the following conclusions:

1. The growth hormones found in abundance not only in the phloem and cambium, but also in the xylem of the stem bundle of pumpkin are formed, partly at least, with the aid of the chloroplasts contained in the living cells of the bundle. This may account for the fact that grafts of *Cucurbitaceae* show a quicker and better growth if they are kept from the very beginning in light, the point of grafting being unobscured.

2. The dead elements of the xylem, the vessels which in this particular instance are surrounded by cells rich in growth substances, participate in some way in the transportation of these substances ([5],[6]).

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REFERENCES

[1] M. Moissejewa. C. R. Acad. Sci. URSS, LXVI (1945). [2] Fr. Meyer, Über die Verteilung des Wuchsstoffes in der Pflanze während Entwicklung, Diss., 1936. [3] Н. Холодный, Фитогормоны, Киев, 1939. [4] М. Мойсеева, Наукова конф. КДУ, в. 1 (1945) (тезисы). [5] A. E. Hitchcock and P. W. Zimmerman, Contr. Boyce Thompson Inst., 7 (1935). [6] G. Bergström, The Transverse Reactions of Plants, Lund, 1939.

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PHYSIOLOGY

A CONTRIBUTION TO THE KNOWLEDGE OF CHEMICAL RELATIONS BETWEEN PLANTS AND ANIMALS

By E. F. KISSELEVA

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Thanks to the evolutional doctrine of Charles Darwin, a wealth of observational data concerning the mutual relations between plants and animals have been accumulated both by botanists and zoologists. The last century was marked by many important discoveries that shed revealing light upon the rôle of bacteria in the life of plants and animals, the regularities underlying the life of soils, forests and other vegetable communities; among these discoveries belongs in the first place that of the «cosmic rôle of plants», deciphered by Timiriazev and others.

Indeed, there is scarcely any biological science that would treat a definite problem without having recourse to the data secured by the study of evolution, or without touching upon the interrelations between animals and plants in their manifold aspect. Some of the studies that have appeared within recent years make us attach increasing interest to the direct chemical relations between plants and animals.

We do not mean here the general problems connected with their mutual relations, so far as nutrition, respiration, *etc.* are concerned, but those involved in the protective apparatus of the various plants, which are intended to defend them against the injurious action of certain animals, or else, the adaptation of animals to definite vegetable organisms. The present author has been prompted to the study of these problems by the stimulating investigations carried on by Tokin and his co-workers, bearing upon the so-called phytoncides ([1],[2]).

We will not discuss here any debatable question involved in the theory of phytoncides, the development of which is still in progress, neither will we dwell upon the rôle of phytoncides in Nature; we wish merely to point out that the results already obtained in this field permit of an entirely novel approach of certain ecological and evolutional problems.

The present author took interest in this problem on account of the possibility of a preferential, and occasionally even exclusive existence of a given insect species under the conditions of a definite plant association. It might well appear that, along with such decisive factors as food, temperature, soil conditions and many others, the production by plants of phytoncides as a means of protection (B. P. Tokin) has also direct bearing upon the fact that certain insects are invariably confined to definite plant species. This note contains a brief account of the data already obtained, which may be of interest at this juncture.

As experimental material served gnats (g. *Aëdes*), midges (*Simulium* sp.), horse-flies (*Chrysops caecutiens* L., *Haematopoda pluvialis* L.) and the common fly (*Musca domestica* L.). Tested were the volatile fractions of bird-cherry (*Padus racemosa* Gilib.), black currant (*Ribes nigrum* L.), white poplar (*Populus alba* L.), maple (*Acer negundo* L.), birch (*Betula verrucosa* Ehrh.), mountain-ash (*Sorbus*

aucuparia L.), yellow acacia (*Caragana arborescens* L.), Siberian apple-tree (*Malus baccata* Borch.), raspberry (*Rubus idaeus* L.).

The experiments carried out under laboratory conditions were arranged according to the following scheme.

In order to obtain a wider area of evaporation, various organs of the plants tested were reduced to a paste; the latter was spread over the bottom of a glass jar or a test-tube of a large diameter. The insects to be tested were placed in the same containers.

For observations carried out under natural conditions the following simple procedure was adopted. A small box provided with a gauze-net (15 × 9 cm), containing the insects under experiment, was fixed for a definite period of time among the branches of a corresponding tree, or shrub.

Of particular interest are the results obtained in our experiments with bird-cherry (see table).

Date	Time elapsed between sampling and experiment	Plant organ tested	Insect	Exposure in min.	Number of insects	Result	Place of experiment
20.V.1944	12 hrs	Leaves	Gnats	3	20	Death	—
	48	»	»	10—15	15	Death after 3 min. exposure	
	48	Blossoms	»	10—15	15		
22.V.1944	Freshly collected	Leaves	»	2	7	Death	Grove
23.V.1944	40 hrs	»	»	10	15	Death after 3—4 min. exposure	Laboratory
	Freshly collected	»	»	2	20	Death	Do
26.V.1944	30 min.	»	Common flies	12	5	Death after 5 min. exposure	»
18.VII.1944	Freshly collected	»	»	10	5	Death	»
	Do	»	Midges	1	Many	Death 54 sec. after exposure	Wood
	»	»	*Haematopoda pluvialis* L.	5—10	5	Death after 5 min. exposure	»
19.VII.1944	Freshly collected	»	*Chrysops caecutiens* L.	5—10	6	Death after 3—4 min. exposure	»
24.VII.1944	Do	»	Midges	2	Many	Death	»
26.VII.1944	»	»	Common flies	10	7	»	Laboratory

Similar results were obtained in the case of field experiments.

For the time being we do not venture any conclusion from the observations here described; we merely wish to point out the unquestionalbe effect of the volatile fraction of the phytoncides produced by the plants upon the insects taken under experiment.

The regularities that underly their relations seem to be rather complicate ones. Suffice it to recollect Tokin's data on *Drosophila melanogaster* which was found to be practically invulnerable by the phytoncides of such plants as onions and garlic. In the course of our investigation we are likely to confront more than once similar specific adaptations of certain insects to definite plants.

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REFERENCES

[1] Б. П. Токин. Бактерициды растительного происхождения (фитонциды). 1942.
[2] Фитонциды. Сборник научных исследований об антисептиках растительного происхождения под ред. С. П. Карпова и Б. П. Токина, Томск, 1944.

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1945. Volume XLIX, № 9

EXPERIMENTAL MORPHOLOGY

CORRELATION OF GROWTH AND DIFFERENTIATION DURING THE POST-EMBRYONIC DEVELOPMENT OF THE PIGEON

By N. V. BELSKY

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The differentiation of a developing organism is known to be accompanied by a decrease in the intensity of its growth. This connexion between growth and differentiation in the course of development seems to be inherent in all multicellular animals, forming one of the most general principles of development. This principle was primarily established in regard to embryonic development, in the course of which the correlation between growth and differentiation is most clearly manifested. It has been established that during periods of particularly intense morphological and histological differentiation of the embryo its growth rate is greatly reduced, while during periods of intense growth no essential changes occur in the structure of the embryo (1).

At the post-embryonic stages of development the intensity of both growth and differentiation is diminished. The question arises, whether the principle under consideration applies also to these later stages of ontogenesis.

Birds represent a suitable object for the study of this problem. The great expenditure of plastic material suffered by the bird in connexion with the development of its plumage should necessarily involve changes in the relations in the growing organism, which are likely to alter essentially the total growth of the organism. Indeed, it has earlier been(2,3) demonstrated that the decrease of the rate of growth in ducks and geese coincides with the period of the most intense growth of feathers in these birds. The period of intense moulting corresponds to a second drop in the rate of growth. Considering that the development of the plumage itself is composed of two processes (growth proper and differentiation), it should be admitted that the coefficient of differentiation adopted in the article cited above was not sufficiently precise.

On the other hand, it seems of importance to study the problem under consideration on other birds, as well. The pigeon is a suitable object, for its post-embryonic growth is most intense and finishes almost completely by the end of the first month of life. At the same time its development of plumage is also characterized by a high rate, nearly the total surface of its body being involved in this process simultaneously.

Our observations include a considerable number of carrier-pigeons. The growth was kept under control by of daily weight records of the young from the moment of hatching to the age of 30 days. Since the development of plumage in pigeons is practically simultaneous in all the pterylia, observation of one of them should suffice for the study of the formation of the bird's feathering. We selected for our studies the flight-feathers of the wing. The total length of the feathers and the length of the unfurling vane were measured simultaneously with the weighing of the young birds. The time of the appearing of the vane and of a change in its length were recorded separately for the purpose of discerning between the growth proper of the feather and its differentiation.

The data thus obtained were used for the calculation of the rate and constant of growth according to the method proposed by Schmalhausen.

As one can see from the table, the average weight of the birds increases from 16 g—immediately after hatching—to 350 g at the age of 30 days. These changes in the average weight of the birds proceed according to an S-shaped curve. Our calculations showed that the weight-increase expressed in per cent (figures not included in the table) drops abruptly beginning with the age of 4 days.

Constant of Growth and Development of Feathering in the Pigeon

Age, days	Average weight, g	*Co. t.*	Average value of constant of growth, *K*	Average length of 4th flight-feather, mm	Average length of vane, mm
0.5	16.7	—		—	—
2.5	37.6	1.218		—	—
4.5	70.3	1.565		—	—
6.5	105.9	1.435	1.336	4.5	—
8.5	142.3	1.327		12.6	—
10.5	179.1	1.265		24.7	0.4
12.5	215.8	1.209		35.9	6.9
14.5	239.2	0.772		49.4	18.4
16.5	253.6	0.493		61.5	31.8
18.5	278.7	0.893	0.707	71.5	40.2
20.5	299.1	0.745		84.5	56.3
22.5	320.9	0.805		96.8	69.5
24.5	334.9	0.537		107.5	84.6
26.5	340.3	0.210		117.9	98.9
28.5	348.5	0.338	0.219	127.7	112.8
30.5	351.0	0.108		132.0	119.0

The character of growth of the birds is, however, more clearly characterized by the true rate of growth and the constant of growth. The change in the value of this constant (see table) shows that the post-embryonic growth of the pigeon is divided into three distinct periods, *i. e.* the period from the 1st to the 12th day, characterized by a relatively high value of the constant, averaging 1.3; a second period from the 12th to the 24th day, with a constant nearly one half of that of the first, averaging 0.7; and a third period characterized by the least value of the constant, averaging only 0.2 and showing that the growth stops completely at this stage. The existence of these periods indicates a lack of homogeneity in the process of the post-embryonic development of the birds under study. It should be surmised that the regular decrease in the energy of growth within the limits of each period and particularly its abrupt drop at the transition to the next period are induced by changes in the development conditions of the organism. On the ground of what has been said above we shall discuss in the present article the changes of the internal conditions connected with the development of the plumage.

The growth of feathers in the pigeon begins on the 6th day after it is hatched, when the rudiments («stumps») of feathers appear on most pterylia, supplanting the embryonic down covering.

On the right side of the table we have reproduced the age changes in the total length of the developing flight-feather, and in a separate column the changes in the length of the vane. As may be gathered from these data, the growth of the rudimentary feathers appearing at the age of 5—6 days is at first most intense, but later on their energy of growth sinks rapidly. Indeed, the relative increase (not included into the table) by the 10th day of the bird's age is only one third of the initial. The differentiation of the feather, as one can judge from the unfurling of the vane, only begins at this time, but from the 12th day onwards an intense

unfurling and increase of the vane takes place. It is important to note that the increase in the size of the vane proceeds along with the fall in the increase of the total length of the feather, which is corroborated by the changes in the value of the constant of growth of the latter. Its average value during the first 10 days is 2.10, while after the 10th day it sinks to one half of this figure (1. 2). Consequently, the decrease in the constant of growth occurs somewhat earlier (by two days) than the period of intense unfurling of the vane. This is bound with the fact that the process of differentiation of the growing feather begins actually a little before the appearance of the vane.

We thus see that the formation of the pigeon's plumage takes place between the 6th and 30th days of its life. An intense growth of the feathers accompanied with but a slight differentiation continues till the 12th day; after this their formation and differentiation became still more intense.

Comparing the data on the development of the feathering with those on the weight and the constant of growth (see table), we can see that a more intense growth of the feathers at the early period (between 6 and 12 days) does not affect the general growth. It should be assumed that the growth of feathers at this period is only one of the items of the sum total of the general growth of the organism. It is not until the age of 12 days, at which the process of differentiation begins to be more sharply expressed, that an abrupt drop in the value of the growth constant takes place.

Thus, the present study demonstrates that the principle of inverse correlation between growth and differentiation indicated above and already established for the embryonic stage of development, holds true in the post-embryonic period of ontogenesis, as well. We must note, however, that the problem of the mechanism of the correlation between growth and differentiation is not considered in the present article. It may prove possible that this mechanism in the post-embryonic period is essentially different from the one which has been found to exist in the embryonic period.

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REFERENCES

[1] И. И. Шмальгаузен (ред.), Рост животных, 1935. [2] Г. Г. Штрайх и Е. А. Светозаров. Усп. зоот. наук. I (1935). [3] Г. Г. Штрайх и Е. А. Светозаров, там же, II (1936).

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EXPERIMENTAL MORPHOLOGY

INTERRELATIONS BETWEEN THE INDUCTIONS OF EMBRYONIC TYPE AND ENDOCRINE DEVELOPMENTAL FACTORS

By V. V. POPOV

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Having established in the case of the eye ([1-5]) the capacity of inductors of the embryonic type to exert their formative action up to the latest stages of ontogenesis, the author was confronted with the problem of ascertaining the interrelations between these inductors and the endocrine development factors prevailing in late ontogenesis. Although some earlier investigators did to a certain degree inquire into that most intricate problem([6-8]), it has so far received no experimental elucidation. The present experiments are the first attempt made in this direction.

To clear up the problem referred to above in an experimental way various modes of attack might be used. We chose one of them, reducing our task to the demonstration of the relative energy of the two kinds of formative effects. It was necessary to find such complexes of cellular materials as could be simultaneously affected both by the inductors of embryonic type, and the endocrine formative factors.

We believe the tadpole's tail skin and opercular membrane, as well as the epidermis covering the main branch of gills and the swimming fringe of the newt larvae, to represent a well-suited test material. In virtue of their histological nature those larval parts would be expected to follow the path of a progressive form production under the influence of embryonic inductors and, moreover, to be capable to resorption due to the effect of an endocrine agent, *viz.* thyroxin, a fact well known since long. It was the ascertaining of the fate of the above cellular materials submitted to the action of inductors, embryonic in type (eye, tympanic cartilage), that made the purpose of the investigations described in the present note.

Work along this line has been carried on in this laboratory since 1939, but the chief idea and the first experiment only (formation of the retina from the tail skin) were published as an autoreport([9]). The methods and results of other experiments repeatedly communicated (colloquium at the chair of embryology, Moscow State University, 1943; Society of Naturalists, 1941; Scientific council of the Zoological Institute, Moscow State University, 1945, and many others) appear in the press for the first time.

1. Formation of the Cornea from Resorbing Skin. Grafts of the tadpole tail skin in place of the external cornea were undertaken with hosts and donors of a similar age. In general, the age of test pairs varied from the young tadpole stage to that of animals in the process of metamorphosis, with their tails slightly shortened. We were particularly interested in late stages at which the formative effect of thyroxin began to be manifest. Control experiments consisted in identical transplantations of the same cellular material, but on to the head and back instead of the region influenced by the eye.

It was found that in controls the transplanted pieces became invariably resorbed approximately toward the end of the metamorphosis. On the contrary, in

the main experiments, despite a deliberate increase in the endocrine component either by adding thyroxin to the medium, or by implanting into test animals small fragments of an active thyroid gland, the latter have never been resorbed, being sooner or later invariably transformed into the cornea. The persistence and the change of the transplant just referred to may be tested on the ground of hetero- and xenoplastic data, as well as by special grafts of marked skin.

2. Induction of the Lens from the Resorbing Epidermis. This work is composed of two parts: 1) experiments on *Anura*, and 2) those on *Urodela*.

The tail epidermis of rather young tadpoles of various species was implanted into the eye cavity of late embryos and tadpoles of various age to replace their previously removed lenses. The implant never failed to become a lens, undergoing no subsequent resorption even at the culmination of metamorphosis, *i. e.* during the highest activity of the thyroid gland. Contrariwise, in controls similar to those of the preceding investigation, the same cellular material was invariably resorbed at the end of metamorphosis.

In *Urodela* (newts) the epidermal cover of gills and swimming fringe of young larvae, *i. e.* an analogous cellular material disappearing in metamorphosis, was used as an implant. The results proved in this case to be similar to those for Batrachians, which holds true for the gill epidermis known to be of entodermal origin. The control was identical with that of the preceding sets. The persistence of the implants, namely their transformation into lens, may be seen from their marking and a successive fixation of test-animals, practised at definite short time intervals. This method made it possible to follow up the course of alterations in the materials implanted. In the experiments here described an increase in the endocrine component was brought about by various manners, transplantations of small parts of an active thyroid into the anterior chamber of the operated eye being one of them.

3. Destiny of the Opercular Membrane Transplanted into the Eye. It is no longer questioned that the orifice of the opercular membrane of the Batrachians undergoing metamorphosis, through which one of the fore legs protrudes, is generally formed under the influence of a hormone, *viz.* of the thyroxin.

Transplantations of the opercular membrane to tadpoles of various species in place of their external cornea, consequently into the sphere of the eye's inductive action, have in all cases resulted in the transplant becoming a new cornea. The above said refers to the outer layer of the transplanted membrane, its inner layer getting always resorbed in the situation indicated. An orifice proper to the opercular membrane has never been formed in the transplant. Control experiments have, on the other hand, shown that in the opercular membrane grafted to the tadpole's back the opening is regularly seen to develop during metamorphosis in a certain percentage of cases.

4. Formation of the Tympanic Membrane from Resorbing Skin. The tympanic membrane of the Batrachians arises at the end of metamorphosis, the skin, according to Helff's data[10], being changed into the ear-drum under the formative action of the ring-shaped tympanic cartilage. The grafting of the tail skin, resorbed in metamorphosis, to the young of the year belonging to different species on to the spot of their ear-drum previously removed, was followed by a regular conversion of the transplant into a new tympanic membrane. In these fundamental experiments in contrast to controls the transplant has never been resorbed. In all instances its fate was easily traceable in connexion with hetero- and xenoplastic transplantations, as well as with those of marked skin. Control experiments did in no way differ from those of the previous investigations.

In all experiments here considered the same phenomenon was invariably observed, *viz.* a local prevalence of embryonic inductors over endocrine factors. Despite an absolute dominance of the latter in late ontogenesis, they do not affect the fate of cellular materials which are found to be in the sphere of the formative influence

of embryonic inductors. These seem to attain a state of integrity at the expense of the «reacting system» not only in a typical embryogenesis ([11], [12]), but at any stage of ontogenesis as well ([1–5], [13]). This fact serves probably to account for the remarkable energy of such inductive effects as from an historical point of view were likewise the most ancient.

From certain considerations, upon which we cannot dwell for lack of space, we were intentionally increasing the endocrine component in all our experiments. This method has, however, in no way affected their results, and the prevalent local significance of embryonic inductors proved to be fully conspicuous in those cases, too. The actual increase in the endocrine factor was clearly shown by a considerably accelerated resolution of the control transplant, when the latter was grafted either on to the animal's head or back under an unrestricted action of thyroxin, without any synchronous inductive influence of the eye or tympanic cartilage.

There seems to be no doubt concerning the most important results of our experiments, namely the demonstration of the persistence of tissues transplanted, *i. e.* their conversion into new structures. The significance of this experimental result is quite evident. As shown by controls, the transplants were able to receive the action of thyroxin. But, if they had been slowly and imperceptibly resolved to be replaced in the same imperceptible manner by the host's skin, this would essentially alter the meaning of our study.

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REFERENCES

[1] V. V. Popov and V. S. Popov, C. R. Acad. Sci. URSS, XVI, No. 4 (1937). [2] В. В. Попов при участии В. С. Попова, Тр. Ин-та экспер. морфогенеза, 6, 65 (1938). [3] В. В. Попов, Уч. зап. Горьк. гос. ун-та, 8, 22 (1938). [4] V. V. Popov, C. R. Acad. Sci. URSS, XXIV, No. 7 (1939). [5] В. В. Попов, Тр. Ин-та экспер. морфогенеза, 7, 27 (1939). [6] И. И. Шмальгаузен, Основы сравн. анатомии, М. 1935. [7] И. И. Шмальгаузен, Организм как целое, М., 1938. [8] И. И. Шмальгаузен, Журн. общ. биол., 1, 349 (1940). [9] В. В. Попов, Тр. каф. гистол. и эмбриол. ГГУ, 1, 37 (1940). [10] O. M. Helff, Physiol. Zool., 1, 89 (1928). [11] Д. П. Филатов, Усп. совр. биол., 3, 440 (1934). [12] O. Mangold, Naturwiss., 22/24, 371 (1932). [13] V. V. Popov and G. Kh. Bimbad, C. R. Acad. Sci. URSS, LI, No. 1 (1946).

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HELMINTHOLOGY

A NEW REARRANGEMENT OF THE TAXONOMY OF THE NEMATODES BELONGING TO THE FAMILY *FILARIIDAE* COBBOLD, 1864

By K. I. SKRJABIN, Member of the Academy, and N. P. SHIKHOBALOVA

A morphological study of the nematodes of the *Filariidae* family enabled Skrjabin and Shikhobalova to suggest in 1936 a new taxonomy of this group of helminths. The taxonomic system developed by the authors was based upon the principle of a correlation between the structure of the cephalic end and the circumoral ornamentation of the *Filariae*, on the one hand, and the structure of the spicules and their size, on the other.

As a result, the *Filariidae* family has been subdivided into 6 subfamilies: 1) *Filariinae* Stiles, 1907; 2) *Diplotriaeninae* Skrjabin, 1915; 3) *Aproctinae* Yorke et Maplestone, 1926; 4) *Setariinae* Yorke et Maplestone, 1926; 5) *Onchocercinae* Leiper, 1911, and 6) *Tetracheilonematinae* Wehr, 1935.

Taxonomy of *Filariae* according to Wehr (1935)

The system developed by the present authors in 1936 differs in principle from the taxonomy of *Filariae* as suggested by Wehr in 1935. According to the American scientist, the *Filariidae* are classed among 4 families: the old family *Filariidae* Cobbold, 1864, and three new ones, founded by this author, *viz. Desmidocercidae*, *Dipetalonematidae* and *Stephanofilariidae*.

We cannot subscribe to the classification suggested by Wehr, in the first place because the four families differentiated by this author are based upon a number of casual characters, without any account of their mutual correlation, and without due analysis of the anatomical characters, some of which are of primary and others of secondary taxonomic importance. In other words, the system advanced by Wehr is entirely destitute of any leading principle, without which the taxonomy of any group of animals loses its scientific value.

Let us illustrate the above-said by the following example. The *Filariidae* family which was separated by Wehr into four families embraces genera with a highly diversified morphology. We encounter here nematodes with or without labria, with epaulette-like formations or without them, with similar, or dissimilar spicules which, in turn, are characterized either by a similar, or dissimilar structure, *etc.* In the old system, according to which all *Filariidae* had been united into a single family, *viz.* the *Filariidae*, the assemblage of so greatly diversified characters within a single family was justifiable. But now that this family has been divided by Wehr into four independent families, the union of diversified characters in a single family becomes a fallacious procedure.

Let us cite a second example. The *Filariidae* family is differentiated by Wehr from the *Dipetalonematidae* family created by this author according to the subdivision of the oesophagus into sections, and the morphology of the larvae of the

first stage. By doing so, Wehr utterly ignores such elements as the circumoral pattern and the chitinous organs of the male sexual system, which play a predominant rôle in the phylogenesis of nematodes. To undervaluate their importance as fundamental taxonomic coefficients would certainly be a mistake.

A New Rearrangement of the Taxonomy of *Filariae*

About a decade elapsed since the taxonomy of *Filariidae* had been revised by Wehr (1935) and by Skrjabin and Shikhobalova (1936). The stock of information available has become much more extensive, yet no detailed revision of the abundant material collected has been ventured up to the present time. At this juncture we attempted a new rearrangement of the taxonomy of this interesting group of nematodes.

The first question we found ourselves to face was, What are the fundamental, most typical structural characters common to all representatives of *Filariidae* and apt to form a basis for the new taxonomy of these nematodes?

All the genera of *Filariidae* hitherto described may be distinguished from one another according to the following main characters:

1) Within the one group of genera the two spicules show an identical structure and are similar, or almost similar in size. The other group of *Filariidae* is distinguished by a sharply different size of spicules which, moreover, differ essentially as to their structure.

2) Within the one group of *Filariidae* the cephalic body-end shows a rather complicate structure, their oral aperture being surrounded either with a peribucal chitinous ring, or with a lateral epaulette-like raised pattern, or else supplied with a crown of spike-like teeth. The other group of *Filariidae* is, on the contrary, characterized by a lack of any complicated patterns ornamenting their cephalic body-end; the latter bears but a simple oral aperture surrounded merely with usual neuro-papillary elements.

3) In spite of the fact that the peculiar features of the chitinous organs, of the male sexual system and of the structure of adoral elements may appear in most unexpected combinations in individual *Filariidae* genera, all these traits are nevertheless dominant, leading characters, and a definite regularity can always be traced in their correlative combinations.

In this connexion it seemed expedient, when attempting to a rearrangement of the taxonomy of the *Filariidae*, to raise these characters to the rank of taxonomic coefficients of familial importance, instead of the subfamilial rank they had occupied in the system arranged in 1936. In other words, we find it necessary to raise the former subfamilies *Aproctinae* and *Setariinae* to the rank of independent families.

Aproctidae nov. fam. will be characterized by a combination of spicules similar both in size and structure with a cephalic body-end lacking any ornamentation; unlike this, *Setariidae* nov. fam. will be characterized by a combination of spicules differing both in size and structure with a cephalic body-end with a rather complicated ornamentation.

The old family is thus reduced in volume by eliminating from it several genera raised by us to the rank of independent familial components; the diagnosis of our third family, *Filariidae*, in its new interpretation will fully coincide with that of the former *Filariinae* subfamily. In other words, it will comprise such genera in which the dimorphous structure of the spicules is combined with the absence of any chitinous or epaulette-like ornamentation at the cephalic body-end.

Along with representatives of the old *Filariinae* family, the characters just enumerated served to typify the representatives of *Onchocercinae* and *Diplotriaeninae*, the former differing from the *Filariinae* in the structure of their cuticle, and the latter, in the presence of a chitinous trident. At the present time we find it necessary to discard the subfamily *Onchocercinae* as a taxonomic unit, and to distribute the genera it comprised between *Filariinae* and *Aproctinae*, in conformity

with their respective characters. As to *Diplotriaeninae*, we find it expedient to refer it as an independent subfamily to *Filariidae*, for we fail to discover any external ornamentation of the cephalic end of their bodies, while their spicules differ both in size and structure.

Whilst analysing the generic components of the newly founded *Setariidae* family, we are at once confronted with the following difficulty. The chitinized organs of the male sexual system in representatives of this family are more or less uniform (spicules differing both in size and structure), whereas their oral elements display an extreme variety of structure. By this circumstance we are induced to split the *Setariidae* family according to the nature of their peribucal organs into three subfamilies: 1) *Setariinae* Yorke et Maplestone, 1926, whose oral aperture is surrounded with a peribucal chitinous ring, liable to assume the shape of labria; 2) *Dipetalonematinae* nov. subfam., with an epaulette-like structure differently pronounced at either side of the oral aperture; 3) *Stephanofilariinae* nov. subfam., with an oral aperture encircled by chitinous indentations.

The third family founded by the present authors, the *Aproctidae*, is characterized by a correlation between their spicules that are similar both in size and structure, on the one hand, and their cephalic end, lacking any external chitinous ornamentation, on the other.

In so far as to this family are referrable both such genera as are entirely lacking any chitinous ring, and such whose rings which invest their oral capsules from inside are but poorly expressed, the *Aproctidae* should naturally be subdivided into 2 subfamilies: *Aproctinae* Yorke and Maplestone, 1926, destitute of a chitinous ring, and *Tetracheilonematinae* Wehr, 1935, with a chitinized oral capsule.

All the families listed above will naturally compose a suborder *Filariata* Skrjabin, 1915, whose revised diagnosis will correspond with the diagnosis of *Filariidae* as conceived in 1936.

On account of what has been said above the following reformed system of the nematodes of the suborder *Filariata* Skrjabin, 1915, is suggested:

A. The Family *Filariidae* Cobbold, 1864, is subdivided into: 1) *Filariinae* Stiles, 1907, and 2) *Diplotriaeninae* Skrjabin, 1915.

B. The family *Setariidae* nov. fam. is subdivided into: 1) *Setariinae* Yorke et Maplestone, 1926; 2) *Dipetalonematinae* nov. subfam., and 3) *Stephanopilariinae* nov. subfam.

C. The family *Aproctidae* nov. fam. is subdivided into: 1) *Aproctinae* Yorke et Maplestone, 1926, and 2) *Tetracheilonematinae* Wehr, 1935.

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REFERENCES

[1] K. I. Skrjabin et N. P. Shikhobalova, Ann. de parasit., XIV, No. 1, 61 (1936). [2] E. Wehr, Proc. Helminth. Soc. of Washington, 2, No. 2, 84 (1935). [3] W. Yorke et P. A. Maplestone, The Nematodes Parasites of Vertebrates, London, 1926, 387.